Homework Chapter 13

1- Find the limit of a_n as n goes to infinity.

a) $a_n = \sqrt{n+a} - \sqrt{n}$ b) $a_n = \ln(\sqrt{n+a}) - \ln(\sqrt{n})$ c) $a_n = \arctan(n)$

d) $a_n = \frac{\ln(n^2)}{n}$ e) $a_n = \frac{Sin(n)}{\sqrt{n}}$ f) $a_n = 2 + (\frac{-2}{\pi})^n$

g) $a_n = (1 - \frac{2}{n})^n$ h) $a_n = \frac{\pi^n}{4^n}$ i) $a_n = n^2 e^{-n}$

2- Show that if the series converge or diverge. If it converges find its sum.

a) $\sum_{1}^{\infty} 3^{-n} 8^{n+1}$ b) $\sum_{1}^{\infty} \frac{1}{5 + 2^{-n}}$ c) $\sum_{1}^{\infty} \frac{n}{\sqrt{1+n^2}}$ d) $\sum_{1}^{\infty} \arctan n$

e) $\sum_{1}^{\infty} \frac{1}{n(n+2)}$ f) $\sum_{1}^{\infty} \ln(\frac{n}{n+1})$ g) $\sum_{1}^{\infty} \frac{1}{2^n} - \frac{1}{2^{n+1}}$

3- Show that if the series (Conditionally /absolutely) Converge or diverge.

a) $\sum_{1}^{\infty} \frac{Sin^2 n}{n\sqrt{n}}$ b) $\sum_{1}^{\infty} \frac{Cos(\frac{\pi n}{6})}{2n^2}$ c) $\sum_{1}^{\infty} \frac{1}{n \ln n}$

d) $\sum_{1}^{\infty} \frac{3}{n(n+3)}$ e) $\sum_{1}^{\infty} \sqrt{\frac{n}{(n+1)^2}}$ f) $\sum_{1}^{\infty} \frac{1+5^n}{2+4^n}$ g) $\sum_{1}^{\infty} \frac{(n+1)!}{n!\ 10^n}$

h) $\sum_{1}^{\infty} \frac{(-1)^n}{(2n)!}$ i) $\sum_{1}^{\infty} \frac{(-1)^n}{\sqrt{2n+1}}$ j) $\sum_{1}^{\infty} \frac{(-1)^n}{\ln(n+1)}$ k) $\sum_{1}^{\infty} \frac{(-1)^n n}{\ln(n)}$

l) $\sum_{1}^{\infty} \frac{(-1)^n 5}{n^3 + 1}$ m) $\sum_{1}^{\infty} \frac{(-100)^n}{n!}$ n) $\sum_{1}^{\infty} \frac{(-1)^n (n^2+3)}{(2n-5)}$ o) $\sum_{1}^{\infty} \frac{(-1)^n \sqrt[3]{n}}{n+1}$

p) $\sum_{1}^{\infty} \frac{(-1)^n}{n\sqrt{\ln(n)}}$ q) $\sum_{1}^{\infty} \frac{n^n}{(-5)^n}$ r) $\sum_{1}^{\infty} \frac{(-1)^n}{(n-4)^2 + 5}$ s) $\sum_{1}^{\infty} \frac{(-1)^{n+1}}{2n+1}$

t) $\sum_{1}^{\infty} \frac{1}{2^n} - \frac{1}{3^{n+1}}$ u) $\sum_{1}^{\infty} n^{\frac{-1}{3}}$ v) $\sum_{1}^{\infty} \frac{n}{1+n^2}$ w) $\sum_{1}^{\infty} n^2 Sin^2(\frac{1}{n})$

x) $\sum_{1}^{\infty} n\ e^{-n^2}$ y) $\sum_{1}^{\infty} \left(\frac{3n+1}{2n-1}\right)^n$ z) $\sum_{1}^{\infty} \frac{n!\ 10^n}{3^n}$ a*) $\sum_{1}^{\infty} \frac{(-1)^{n+1}}{2n+1}$

b*) $\sum_{1}^{\infty} \frac{\ln(n)}{e^n}$ c*) $\sum_{1}^{\infty} \frac{1}{n(\ln n)^2}$ d*) $\sum_{1}^{\infty} \frac{5}{3^n+1}$ e*) $\sum_{1}^{\infty} \frac{1}{5n^2-n}$

f*) $\sum_{1}^{\infty} \frac{Cos(\frac{1}{n})}{n^2}$ g*) $\sum_{1}^{\infty} \frac{n^{\frac{4}{3}}}{8n^2+5n}$ h*) $\sum_{1}^{\infty} \frac{(-1)^n n^3}{e^n}$ j*) $\sum_{1}^{\infty} \frac{(-1)^n n^n}{n!}$

4- Find radius and interval of convergence for each series

a) $\sum_{1}^{\infty} \frac{n(2x-1)^n}{4^n}$ b) $\sum_{1}^{\infty} \frac{(x-4)^n}{n\ 5^n}$ c) $\sum_{1}^{\infty} \frac{2^n(x-3)^n}{(n+3)}$ d) $\sum_{1}^{\infty} \frac{x^n}{n!}$

e) $\sum_{1}^{\infty} \frac{3^n x^n}{n!}$ f) $\sum_{1}^{\infty} \frac{(x-3)^n}{2^n}$ g) $\sum_{1}^{\infty} \frac{(-1)^n(x-4)^n}{(n+1)^2}$ h) $\sum_{1}^{\infty} \frac{(2n+1)!}{n^3}(x-2)^n$

i) $\sum_{1}^{\infty} \frac{(-1)^n(x+1)^n}{n}$ j) $\sum_{1}^{\infty} \frac{5^n x^n}{n^2}$

5- Find the power series representation for each function and determine the radius of convergence.

a) $f(x)=\ln(1+x)$ b) $f(x)=Arc\tan(2x)$ c) $f(x)=\frac{1}{x^2+25}$ d) $f(x)=\frac{1+x^2}{1-x^2}$

6- Approximate the following integrals

a) $\int_{0}^{0.2} \frac{1}{1+x^4}dx$ b) $\int_{0}^{0.5} \arctan(x^2)\ dx$ c) $\int_{0}^{0.5} x^2\ e^{-x^2}dx$ d) $\int_{0}^{0.5} Cos(x^2)\ dx$

e) $\int_{0}^{t} x^2(\arctan(x^2)dx$ f) $\int_{0}^{t} \sqrt[3]{1+x^2}dx$ g) $\int_{0}^{t} x^3\ e^{-x^2}dx$ h) $\int_{0}^{t} \frac{Secx}{x}dx$

i) $\int_{0}^{t} x\ln(1+x)\ dx$ j) $\int_{0}^{t} \frac{Sinx^2}{x}dx$

7- Show that

a) $\sum_{2}^{\infty} \ln(1-\frac{1}{n^2}) = -\ln 2$ b) $\sum_{0}^{\infty} (x-3)^n = \frac{1}{4-x} \quad if \quad 2<x<4$

c) $\sum_{2}^{\infty} \frac{1}{n(\ln(n))^P} \begin{cases} P>1 & Converges \\ P \le 1 & Diverges \end{cases}$ d) $4\sum_{0}^{\infty} (-1)^n \frac{1}{2n+1} = \pi \quad -1 \le x \le 1$

e) $\sum_{0}^{\infty} (-1)^n \frac{1}{1+n} = \ln 2$

8- Find the sum of the following series

a) $\sum_{1}^{\infty} nx^{n-1} \quad \forall |x|<1$ b) $\sum_{1}^{\infty} nx^n \quad \forall |x|<1$ c) $\sum_{1}^{\infty} n(n-1)x^n \quad \forall |x|<1$

d) $\sum_{1}^{\infty} \frac{n}{2^n}$ e) $\sum_{1}^{\infty} \frac{n^2}{2^n}$ f) $\sum_{1}^{\infty} \frac{n^2-n}{2^n}$

9- Use the series to evaluate the limit

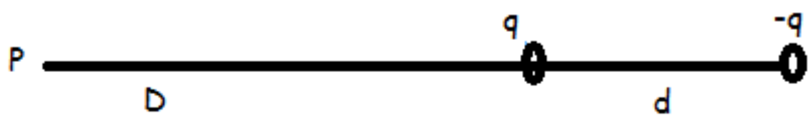
a) $\lim_{x\to 0} \frac{x-\ln(1+x)}{x^2}$ b) $\lim_{x\to 0} \frac{1-Cosx}{1+x-e^x}$ c) $\lim_{x\to 0} \frac{Sinx - x + \frac{1}{6}x^3}{x^5}$

10- Use power series for $Arc\tan x$ to show $\pi = 2\sqrt{3} \sum_{0}^{\infty} \frac{(-1)^n}{(2n+1)3^n}$

11- Show that $\int_{o}^{1/2} \frac{dx}{x^2-x+1} = \frac{\sqrt{3}}{9}\pi$ then use $\frac{1}{x^3+1}$ to show that

$$\pi = \frac{3\sqrt{3}}{4} \sum_{0}^{\infty} (\frac{-1}{8})^n [\frac{2}{3n+1} - \frac{1}{3n+2)}]$$

12- Equal and opposite sign charges a distance d from each other create an Electrical dipole. The electric field at a point P is $E = \frac{q}{D^2} - \frac{q}{(D+d)^2}$



(See the figure)

Use power series of d/D to expand the field and show that $\lim_{D\to\infty} E = \frac{consta}{D^3}$

13- The equation for water wave is modeled as $v^2 = \frac{g\lambda}{2\pi}\tanh(\frac{2\pi d}{\lambda})$. Show that

$$\begin{cases} v \approx \sqrt{\frac{g\lambda}{2\pi}} & (if\ Water\ is\ deep) \\ v \approx \sqrt{gd} & (if\ Water\ is\ Shallow) \end{cases}$$

Where d is the depth of the water.

14- The electric potential V at a point P due to a uniformly charged disk with radius R, surface charge density σ and a distance d from the point is $V = 2\pi k\sigma(\sqrt{d^2+R^2} - d)$. Where K is Coulomb's constant.

Show that $V = \frac{\pi k R^2 \sigma}{d}$ for very large d

15- Prove Taylor expansions