Homework chapter 12

1-For $r(t) =< \frac{2}{3}t^3 + 4t,\ \ t^2,\ \ 3t >$

a) Find, a_T and a_N at t = 2.

b) Find the osculating circle at t = 2.

c) What happens to the curvature as t goes to infinity?

2- For $r(t) =< t, \frac{\sqrt{6}}{2}t^2, t^3 >$

a) Find the tangential and normal component of acceleration

b) Find the osculating circle at t = 2.

3- Prove the formula for curvature in rectangular form and find the curvature of $f(x) = \tan x - x$ at $x = \frac{\pi}{6}$

4- a) Find radius of curvature on $r(t) =< Cos t,\ \ S\text{int},\ \ \frac{1}{2}t^2 >$

b) Show that the radius is increasing at all time (except at $t = 1$).

5- Verify that for $r(t) =< Cos t,\ \ S\text{int},\ \ t >$ all the unit vectors $T(t),\ \ N(t),\ \ and\ \ B(t)$ are orthogonal to each other.

6- Prove the formula for curvature in polar forms and find the curvature of $r = Cos(2\theta)$

7- Show that $\frac{dN}{ds} = \tau\ B - \kappa\ T$

8- Prove that $\ddot{r} = \ddot{s}\ T + \kappa(\dot{s})^2 N$

9- Prove that $\tau = \frac{(\dot{r} \times \ddot{r}) \bullet \dddot{r}}{\left|\dot{r} \times \ddot{r}\right|^2}$

10- Find the osculating circle to the curve $r(t) = < t, \frac{1}{2}t^2 >$ at time $t = 1\sec$. And Find how fast the area of the circle increases as the time increases.

11- Prove the formula for curvature in parametric forms and find the curvature for $r(t) = < t, \frac{1}{3}t^3 >$ at $t = 1\sec$

12- a)Graph $r_1(t) = < t, t^2 >$ and $r_2(t) = < t, t^3 >$. Plot points (Use $t = -2, -1, 0, 1, 2$)

b) Find $T(t)$, $N(t)$ and $B(t)$ for each vector functions.

c) Using $B(t)$ from above to conclude that there is no twisting in the vector functions, therefore easily one can find $N(t)$ from $T(t)$ without using any calculus.

d) Draw $N(t)$ and $T(t)$ in graph of part (a) for $t = -2, -1, 0, 1, 2$ sec

13- a) Graph $r_3(t) = < t, t^2, t^3 >$. Plot points (Use $t = -2, -1, 0, 1, 2$)

b) Show that $T(t) = \frac{< 1, 2t, 3t^2 >}{\sqrt{Something}}$ and $N(t) = \frac{< -4t - 18t^3, 2 - 18t^4, 6t + 12t^3 >}{\sqrt{Something}}$

c) Find $B(t)$.

14- Show that if any ray parallel to axis of a parabola bounces off the parabola, it passes through the focal point of the parabola $r(t) = < t, ct^2 >$

15- Show that if any ray originates from one focal point of an ellipse, after it bounces off the ellipse, it passes through the other focal point of the ellipse $r(t) = < a\,Cost, b\,\sin t >$

16- Show that if any ray originates from one focal point of a hyperbola, after it penetrates through the hyperbola, it passes through the other focal point of the hyperbola. $r(t) = < a\,Sect, b\,Tant >$ or $r(t) = < a\,Cosht, b\,Sinht >$

17- Find the arc length of a curve in 3 and 4 dimensions, and then give some reasoning for the result.

Vector in 3-D space $r(t) = < \cos t, \sin t, t >$ and $|\dot{r}(t)| = \sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2}$

Vector in 4-D space $r(t) = < t, \cos t, \sin t, t >$ and $|\dot{r}(t)| = \sqrt{-1 + \dot{x}^2 + \dot{y}^2 + \dot{z}^2}$

18- Given $\vec{r}(t) = < \frac{1}{2}t^2 - 1,\ 4t >$ a) Graph $\vec{r}(t)$ for $t \geq 0$

b) Draw $\vec{v}(3),\ \vec{a}(3),\ \vec{T}(3),\ \vec{N}(3),\ \vec{B}(3)$

19- Given $\vec{r}(t) = < t,\ 2t\ \ 2t >$, Find $\vec{r}(s)$ and then show that $\kappa = \left|\frac{d\vec{T}}{ds}\right| = 0$

20-Find the arc length of $\vec{r}(t) = < 4t,\ \frac{1}{2}t^2,\ 3t >$, $0 \leq t \leq t^*$.

21- Find $\vec{B}(12)$ For $\vec{r}(t) = < 3t,\ \frac{1}{2}t^2,\ 4t >$

22- A particle at $t = -1$ sec on $\vec{r}(t) = < t,\ \frac{1}{3}t^3 >$ moves with the rate of $|\vec{r}'| = v\ m/s$ and emits a particle in the direction of its velocity. The particle cuts the osculating circle of the curve at $t = 1$ sec in two points. How fast the distance of the points changes in terms of v?