Chapter 11 Partial solution

1- For line p(3,2,-2) and $\vec{V} =< 1,-1,2 >$ and the plane S which contains points A(1,2,1), B(2,-1,2), and C(0,-2,1).

a) Find the point of intersection of the line and the plane. (time and space)

First, we have to find the equation of lines.

We know the point P and the vector V, then we can derive the equation of line;

$$x = 3 + t$$
$$y = 2 - t$$
$$z = -2 + 2t$$

$\overline{AB} =< 1,-3,1 >$ $\overline{BC} =< -2,-1,-1 >$

Where (a,b,c)= $\overline{V} =< 1,-1,2 >$; $< x_1, y_1, z_1 > = p(3,2,-2)$

Normal vector of the plane: $(\overline{AB} \times \overline{BC}) = \begin{vmatrix} i & j & k \\ 1 & -3 & 1 \\ -2 & -1 & -1 \end{vmatrix} = 4i - j - 7k = \langle 4,-1,-7 \rangle$

Then, we can get equation of plane: $4(x-1) - 1(y-2) - 7(z-1) = 0$

Which is $4x - y - 7z + 5 = 0$

Afterwards, we can find parameterize t by plugging line equation to the plane equation:

Thus, $4(3+t) - (2-t) - 7\ (-2+2t)\ + 5 = 0$

Solve for parameterize t , $29 - 9t = 0, \quad t = 29/9$

The value of time is the parameterize T multiply by $|\vec{V}| = \sqrt{6}$

Since we have parameterize t, we can locate the intersection point Q:

$Q = \begin{cases} x(t) = 3 + t = 56/9 \\ y(t) = 2 - t = -11.9 \\ z(t) = -2 + 2t = 40/9 \end{cases}$ Therefore, the point of intersection is $Q =< \frac{59}{9}, \frac{-11}{9}, \frac{40}{9} >$

b) Find the acute angle between the line and the plane

From the equation of plane $(4x - y - 7z + 5 = 0)$

We know that the normal vector of the equation is: $\overline{N} =< 4,-1,7 >$

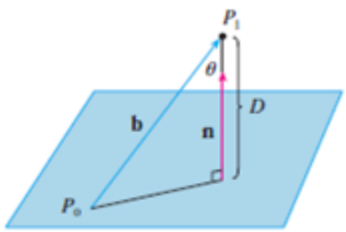
We also have the directional vector of line: $\vec{V} =< 1,-1,2 >$

In order to find the angle between, we have to use the formula:

$$Cos\theta = \frac{\overline{V}\,\overline{N}}{|\overline{V}\,\overline{N}|} = \frac{4+1-14}{\sqrt{66} \times \sqrt{6}} = \frac{-3}{\sqrt{44}}$$

SOLUTION: $\theta = Cos^{-1} \frac{-3}{\sqrt{44}}$

c) How far the position of a particle on the line at $t = 2\sec$ is away from the plane?



Since it takes more than 2 sec to reach the plane, so we can find the particle on line at t=2

$x = 3 + t = 3 + 2 = 5$

$y = 2 - t = 2 - 2 = 0$

$z = -2 + 2t = -2 + 4 = 2$

Then, we will get a point P_1 on the line, which is $P_1\,(5,0,2)$

We also have a point of intersection: $Q = \left\langle \frac{56}{9}, \frac{-11}{9}, \frac{40}{9} \right\rangle$

After we got the pointP_1, we can find the vector b,

which is $P_1Q =< \frac{11}{9}, \frac{-11}{9}, \frac{22}{9} >$

finally, we can use the distance formula of a point to plane:

$$\text{Distance} = Comp_{\vec{N}}^{D\vec{Q}} = \frac{D\vec{Q} \cdot \vec{N}}{|\vec{N}|} = \frac{\left|\frac{11}{9} * 4 + \frac{11}{9} - \frac{22}{9} * 7\right|}{\sqrt{66}} = \frac{96}{\sqrt{66}}$$

d) If the source of light is shines normal to the plane, how fast the shadow of the Particle is moving on the plane?

Speed of the particle $|\vec{V}| = \sqrt{6}$ times $Cos\theta$ from part B

2- For the given Planes $S_1 : 2x + y - z = 4$ and S_2 $x + 2y + z = -1$

a) Find line of intersection of the planes

Solutions: let $z = t$, we can get $\begin{matrix} 2x + y - t = 4 \\ x + 2y + t = -1 \end{matrix}$

Solve this system: $\text{let } x = -1 - t - 2y$ (derive from the second equation)

Solve y in terms of t: $y = -2 - t$ $\begin{matrix} \text{Solve x in terms of t: we get } x = t + 3 \\ \text{Solve z in terms of t: we have known} \end{matrix}$

Solution: Thus, the line equation will be $\begin{matrix} x = t + 3 \\ y = -2 - y \\ z = t \end{matrix}$

b) Find the angle between the planes $S1 : 2x + y - z = 4$ and $S2 : x + 2y + z = -1$

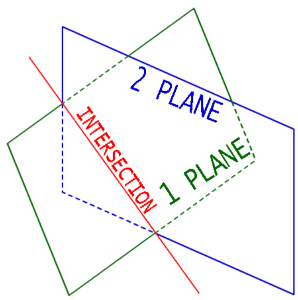
We can get the normal vectors for two planes: $\vec{N}_1 =< 2, 1, -1 > \vec{N}_2 =< 1, 2, 1 >$

Afterwards, we can use the formula to solve the angle:

$$Cos\theta = \frac{\overline{N_2}\,\overline{N_1}}{|\overline{N_2}\,\overline{N_1}|} = \frac{2 + 2 - 1}{\sqrt{6} \times \sqrt{6}} = \frac{1}{2}$$

Solution: $\theta = \cos^{-1}(\frac{1}{2}) = \frac{\pi}{3}$

c) Graph the planes and indicate the line of intersection and the angle



d) If a particle on plane S_1 at point (1, 1,-1) moves directly to pint H on line of intersection of the planes, what are the coordinates of point H?

$P(1,1,-1)$ $H(3+t,-2-t,t)$. $\vec{PH} = <2+t,-3-t,t+1>$ $\quad \vec{N}_1 = <2,1,-1>$

Normal vector $\vec{N}_1$ is perpendicular to the plane S_1. We can get

$\vec{PH} \cdot \vec{N}_1 = 0 \Rightarrow 2(2+t)+(-3-t)-(t+1)=0 \quad t \in R$

Since $t=1$, point $H(4,3,1)$

If the point $P(1,1,-1)$ shines a laser beam in the direction of $\vec{V}(1,-1,2)$ what are the coordinate of point Q intersection of laser beam with plane S_2?

The equation of a laser beam: $\begin{cases} x(t)=1+t \\ y(t)=1-t \\ z(t)=-1+2t \end{cases}$ The equation of plane $S_2: x+2y+z=-1$

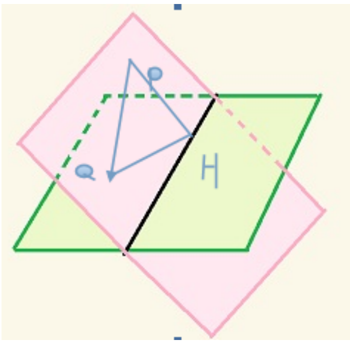
Find the intersection:

- Substitute for variables: $: (1+t)+2(1-t)+(-1+t)=-1$
- Solve parameterize $t=3$
- Solve the equation in order to find the intersection (x,y,z)

$$\text{Point Q} \quad \begin{aligned} x &= 1+t = 1-3 = -2 \\ y &= (1-t) = 1+3 = 4 \\ z &= -1+2t = -1-6 = -7 \end{aligned}$$

Solution: Q(-2,4,-7)

d) Find the Area of a triangle made by point P, Q, and H as its vertices?



$P(1,1,-1)\ \ Q(-2,4,-7)\ H(4,-3,1)$

Find: $\overrightarrow{PQ} = \langle -3,3,-6 \rangle and \overrightarrow{PH} = \langle 3,-4,2 \rangle$

Then find the area: Area of triangle is half of the parallelogram. $A = \frac{1}{2}\left|\overrightarrow{PQ} \times \overrightarrow{PH}\right|$

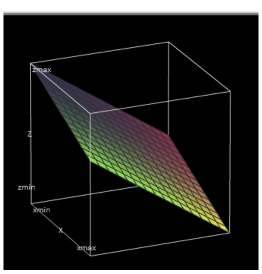
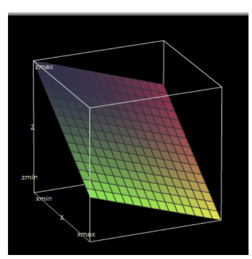
$$P\vec{Q} \times P\vec{H} = \begin{Vmatrix} I & J & K \\ -3 & 3 & -6 \\ 3 & -4 & 2 \end{Vmatrix} = (6-24)I - (-6+18)J + (12-9)K = -18I - 12J + 3K = < -18, -12, 3 >$$

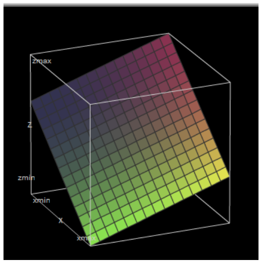
Solution: Thus, Area = $\frac{1}{2}\left|P\vec{Q} \times P\vec{H}\right| = \frac{1}{2}\sqrt{18^2 + 12^2 + 3^2} = \frac{\sqrt{477}}{2}$

3- Solve the systems of equations by finding point of intersection of the planes

$$x + 2y - z = 3$$
$$2x + y + z = 4$$
$$2x - y + 2z = 5$$

First find line of intersection of two planes and then use the line and the third plane to find the point of intersection.

Graphs:  $x + 2y - z = 3$  $2x + y + z = 4$


$2x - y + 2z = 5$

Solve this system, first we have to set z=t

Solve first two equation $x + 2y - z = 3$ and $2x + y + z = 4$set t =z

We get $t = x + 2y - 3$ from the first equation; then we can plug it into second equation: $2x + y + (x + 2y - 3) = 4 \quad 3x + 37 = 7$

We finally get $x = \frac{7}{3} - y$ After we get this, plug it back to $x + 2y - t = 3$

$y = \frac{2}{3} + t$ Then find x in terms of t: $x = \frac{7}{3} - (\frac{2}{3} + t) = \frac{5}{3} - t$ We get the line equation: $\begin{cases} x(t) = (5/3) - t \\ y(t) = (2/3) + t \\ z(t) = t \end{cases}$

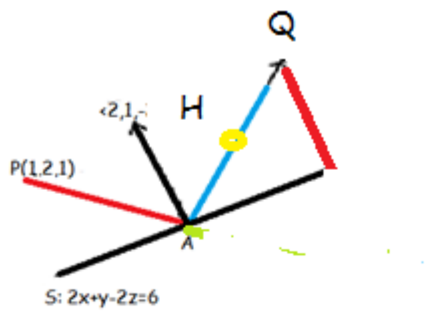
The third plane equation is $2x - y + 2z = 5$

Then, we have to solve for t : $2(\frac{5}{3} - t) - (\frac{2}{3} + t) + 2t = 5$ We get the $t = \frac{-7}{3}$

When plug to find the line of intersection (is $2x - y + 2z = 5$)

Solution: Coordinate of intersecting point is $(x, y, z) = (4, \frac{-5}{3}, \frac{-7}{3})$

4-A Laser beam at point P (1, 2, 1) shines on to mirror S:$2x + y - 2z = 6$ in the direction of $\vec{V} = < 2,2,1 >$. We want to locate position of a detector after 2sec which the beam bounces off of the plane. How far this point is away from point P(1,2,1)?



Because the equation of plane is $2x + y - 2z = 6$, so the vector normal to the plane is $\vec{n} = \langle 2,1,-2 \rangle$ Then we can derive the line(laser) equation of $\overrightarrow{PA}$.

$$\bar{v}* = \bar{v} - 2\,\mathrm{Pr}\,oj_{\bar{n}}^{\bar{v}} \qquad \bar{v}* = <2,2,1> - 2\frac{<2,2,1> \bullet <2,1,-2>}{9} <2,1,-2> = < \frac{-7}{9}, \frac{10}{9}, \frac{25}{9} >$$

We can get the line equation of QQ' $\begin{cases} x(t) = 1 + (2/3)t \\ y(t) = 2 + (2/3)t \\ z(t) = 1 + (1/3)t \end{cases}$ Then find the interaction of line and plane: $2x + y - 2z = 6$ so, $2(1 + \frac{2}{3})\ t + 2 + \frac{2}{3}t - 2 - \frac{2t}{3} = 6 \quad t = 3s$

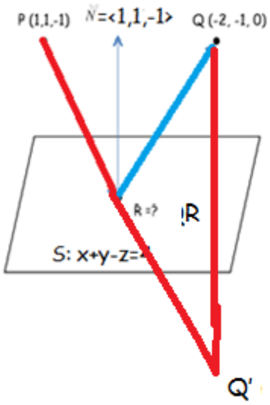
Intersection: $A(1+2, 2+2, 1+1) = (3,4,2)$

Find Line equation of QA $\begin{cases} x = 3 + \frac{-7}{9\sqrt{774}}t \\ y = 4 + \frac{10}{9\sqrt{774}}t \\ z = 2 + \frac{25}{9\sqrt{774}}t \end{cases}$ when t =2

Find lactation of point H the position of a detector after 2sec which the beam bounces off of the plane.

Then use Pythagorean Theorem to find distance between the points.

5- A Laser beam at point P (1, 1, -1) is aimed at point Q (-2, -1, 0) after it bounces off of mirror S: $x + y - z = 4$ What is the location on the mirror which the beam bounces off of?



Directional vector of QQ' is equal to normal vector of the plane, but we have to normalize them in order to have the same unit.

$$\frac{\vec{V}}{|V|}=\frac{\langle 1,1,-1\rangle}{\sqrt{1^2+1^2+1^2}}=\left\langle \frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}},-\frac{1}{\sqrt{3}}\right\rangle$$

We have to drive the line equation QR $\begin{cases} x(t)=-2+(1/\sqrt{3})t \\ y(t)=-1+(1/\sqrt{3})t \\ z(t)=0-(1/\sqrt{3})t \end{cases}$ Then we can find the t

by plugging into plane equation: $x+y-z=4$

$-2+\frac{1}{\sqrt{3}}t-1+\frac{1}{\sqrt{3}}t-\frac{1}{\sqrt{3}}t=4,\ldots\ldots\ldots\ldots t=\frac{7}{\sqrt{3}}$ After that, we can find Q' by doubling the time due to the symmetry

$$x=-2+\frac{1}{\sqrt{3}}\times\frac{7}{\sqrt{3}}=\frac{8}{3}$$

$$y=-1+\frac{1}{\sqrt{3}}\times\frac{7}{\sqrt{3}}=\frac{11}{3}$$ Find PQ' = $\overrightarrow{PQ}=(\frac{8}{3}-1,\frac{11}{3}-1,\frac{-14}{.3}+1)=\left\langle\frac{5}{3},\frac{8}{3},\frac{-11}{3}\right\rangle$

$$z=-\frac{1}{\sqrt{3}}\times\frac{7}{\sqrt{3}}=\frac{-14}{3}$$

Normalize the vector to get the same unit:

$$\frac{\overrightarrow{PQ'}}{|PQ'|}=\frac{\frac{5}{3},\frac{8}{3},\frac{-11}{3}}{\sqrt{(\frac{5}{3})^2+(\frac{8}{3})^2+(\frac{-11}{3})^2}}=\left\langle\sqrt{\frac{210}{3}}\frac{5}{3},\frac{210}{3}\frac{8}{3},\frac{210}{3}\frac{-11}{3}\right\rangle$$

Find line equation PQ' $\begin{cases} x(t)=1+(\sqrt{\frac{210}{3}}\ \frac{5}{3})t \\ y(t)=1+(\sqrt{\frac{210}{3}}\ \frac{8}{3})t \\ z(t)=-1-(\sqrt{\frac{210}{3}}\ \frac{11}{3})t \end{cases}$

Plug in the equation of line in to equation of plane to find t.

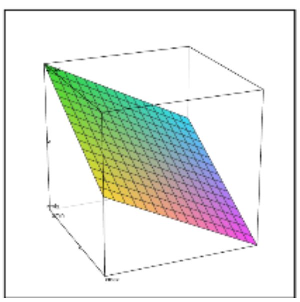
$$\mathrm{x}+y-z=4,$$

$$1+\sqrt{\frac{210}{3}}\frac{5}{3}t+1+,\frac{8}{3}\sqrt{\frac{210}{3}}t-1-\frac{11}{3}\sqrt{\frac{210}{3}}t=4$$

6- Given plane $S_1 : \mathrm{x}+\mathrm{y}+\mathrm{z}=2$, Graph it

a) Find the volume created by plane S and the coordinate planes in the first octane?

b) Find the distance of the plane to the origin.

Graph 

a)

x	y	z
0	0	2
0	2	0
2	0	0

Thus, we got 3 points. a(0,0,2) b(0,2,0) c(2,0,0),

$$Volum=\left|a\bullet(b\times c)\right|=\begin{vmatrix}0 & 0 & 2\\0 & 2 & 0\\2 & 0 & 0\end{vmatrix}=4$$

This is the volume of parallelogram. To find the volume of tetrahedron divide the volume by 6. So the solution is (4/6) =(2/3)

b) First, we have to find point P on the plane $S_1 : x + y + z = 2$;

- We set $x = 0, z = 0$ *then* $y = 2$. Thus, point P will be (0,2,0)

Since we have to find the distance of the plane to the origin, Point Q(0,0,0)

- We have to find vector $\overrightarrow{PQ}$,which is $< 0, -2,0 >$
- Then, from $S_1 : x + y + z = 2$, we know the normal vector of the plane is:$<1,1,1>$

Solution: $D = comp\frac{\overrightarrow{PQ}}{\overline{N}} = \frac{(0 \cdot 1) + (-2) \cdot 1 + 0 \cdot 1}{\sqrt{1+1+1}} = \left| \frac{2\sqrt{3}}{3} \right| = \frac{2\sqrt{3}}{3}$

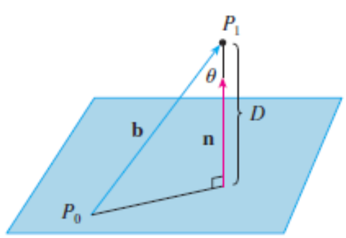
7- Find a general formula for

a) The distance of a point to a plane. (2 different methods)

Method 1:

As you can see the distance D is a component of b, $D = |b| \cos\theta$,

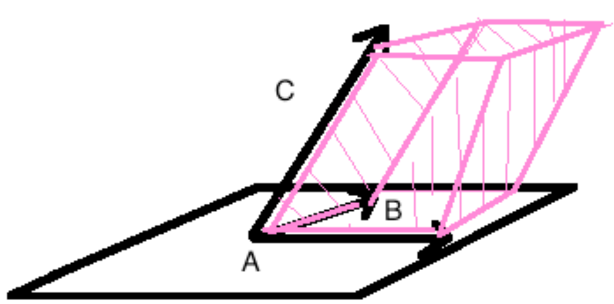
The scalar projection of onto (also called the component of along) is defined to be the signed magnitude of the vector projection, which is the number $|b| \cos\theta$.



$(a \cdot b)=|a||b|\cos\theta$, where $|b|\cos\theta$=D

$(a \bullet b) = |a||b|\cos\theta....\text{where}|b|\cos\theta = D$

This can be denoted $D = comp\frac{b}{a} = \frac{(\overline{a}\overline{b})}{|\overline{a}|}$



Method 2:

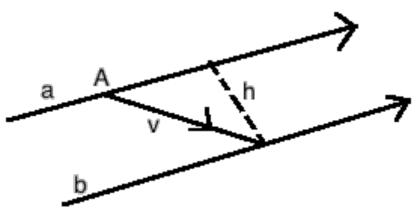
Let h be the shortest distance between the point and the plane. Now we can use the equation of volume of the tetrahedron, which is equal to the area of the tetrahedron multiplied by the height.

$V = A \times h$ Let $V = \bar{c}(\bar{a} \times \bar{b})$ $Volume = \left|\overrightarrow{ab} \times \overrightarrow{ac}\right| h$ Then we get

$\vec{c} \cdot (\vec{a} \times \vec{b}) = \left|\overrightarrow{ab} \times \overrightarrow{ac}\right| h$ Solution: $h = \dfrac{\vec{c}(\vec{a} \times \vec{b})}{\left|\overrightarrow{ab} \times \overrightarrow{ac}\right|}$

b) The distance between two parallel planes. (2 different methods)

Method 1: using the property of vector



Assume that we have two line parametric equation $\vec{a}$ and $\vec{b}$, where $\vec{v}$ is a vector connects to $\vec{a}$ and $\vec{b}$.

$$\vec{a} = \begin{pmatrix} at + d \\ bt + e \\ ct + f \end{pmatrix} \text{ and } \vec{b} = \begin{pmatrix} gs + j \\ hs + k \\ is + l \end{pmatrix}$$

We set t =0 so that A (d,e,f),which connects to $\vec{a}$ and $\vec{b}$

Then, we get $\vec{a}, \vec{b}$ is $\vec{v} = \begin{pmatrix} gs + d \\ hs + e \\ is + f \end{pmatrix}$

The shortest distance between the two vectors is when v is perpendicular to b, which i

$$\vec{v} \cdot \vec{b} = 0 \quad \begin{pmatrix} gs + d \\ hs + e \\ is + f \end{pmatrix}\begin{pmatrix} g \\ h \\ i \end{pmatrix} = 0 \quad \begin{pmatrix} g(gs + d) \\ h(hs + e) \\ i(is + f) \end{pmatrix} = 0 \quad (g^2 + h^2 + i^2)s + (d + e + f) = 0 \quad s = \frac{(d + e + f)}{(g^2 + h^2 + i^2)}$$

$$\vec{v} = \begin{pmatrix} g\dfrac{(d + e + f)}{(g^2 + h^2 + i^2)} + d \\ h\dfrac{(d + e + f)}{(g^2 + h^2 + i^2)} + e \\ i\dfrac{(d + e + f)}{(g^2 + h^2 + i^2)} + f \end{pmatrix}$$

Since we got the line parametric equation, if we want to find the shortest distance, we have to find the magnitude of this line. Basically, we use formula

Solution: find it. $D = |\vec{v}| = \sqrt{a^2 + b^2 + c^2}$.

Method 2:using calculus (first derivative) to find the shortest distance

$$f(t) = |a(t) - (b(0)|^2$$
$$= |(at + d, bt + e, ct + f)|^2$$
$$= (a^2 + b^2 + c^2)(t^2) + (da + eb + cf)t + d^2 + e^2 + f^2$$
$$f'(t) = (a^2 + b^2 + c^2)(t) + (da + eb + cf)$$
$$0 = (a^2 + b^2 + c^2)(t) + (da + eb + cf)$$
$$\frac{-(da + eb + cf)}{(a^2 + b^2 + c^2)} = t$$

'

$$f(t') = (a^2 + b^2 + c^2)\left[\frac{(da + eb + cf)}{(a^2 + b^2 + c^2)}\right]^2 + (da + eb + cf)\left[\frac{(da + eb + cf)}{(a^2 + b^2 + c^2)}\right] + d^2 + e^2 + f^2$$
$$f(t')^2 = \frac{(da + eb + cf)^2}{(a^2 + b^2 + c^2)} + \frac{(da + eb + cf)^2}{(a^2 + b^2 + c^2)} + d^2 + e^2 + f^2$$
$$f(t') = \sqrt{\frac{(da + eb + cf)^2}{(a^2 + b^2 + c^2)} + \frac{(da + eb + cf)^2}{(a^2 + b^2 + c^2)} + d^2 + e^2 + f^2}$$

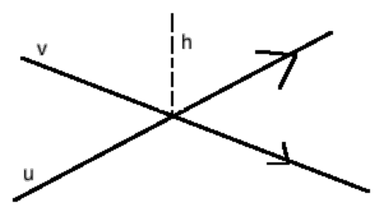
Where f(t') is the shortest distance.

C)The distance between two skewed lines.

-First, we assume that we have to line

$x_1 = at + d, \quad y = bt + e, \; z = ct + f$

$x_2 = gs + j, \quad y = hs + k, \; z = is + l$



Then their directional vectors can be written as

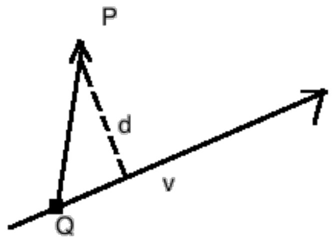
$x_1 : \vec{u} = \langle a, b, c \rangle \quad$ and $x_2 : \vec{v} = \langle g, h, i \rangle$

The shortest distance occurs when $\vec{n} = \vec{u} \times \vec{v}$, which is when 2 lines are perpendicular to each other.

Find a vector in a plane that made with the point R(x,y,z) and t=0 so that P(d,e,f)

Then, $\begin{array}{l}\vec{n}\cdot p\vec{r}=0\\(\vec{u}\times\vec{v})\cdot p\vec{r}=0\end{array}$ the minimum distance would be this plane's values and the point P(d,e,f) plugged into the equation found in part $=\dfrac{|ax_0+by_0+cz_0|}{\sqrt{a^2+b^2+c^2}}$ solution

D) The distance between a point and a line. (4 different methods)



Method 1: using the property of vector

Assuming that we have a line, $\vec{v}$, and a point on that line Q, forms another vector with an point, P, then apply the property of vector:

$$|\mathrm{d}|=\sqrt{|\overline{QP}|^2-\left|comp\,\frac{\overline{QP}}{\overline{V}}\right|^2}$$

Method 2: Applying calculus

$$\begin{aligned}x&=a+ct\\y&=b+dt\end{aligned}$$ and a point P=(x_0, y_0)

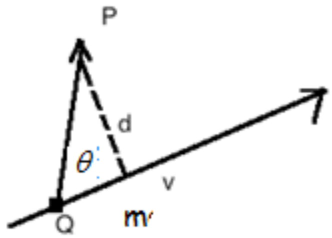
Then, differentiate and find optimization, which is the shortest distance between the point and the line.

$$d^2=(a+ct)^2+(b+dt)^2$$
$$d(t)=a^2+2act+c^2t^2+b^2+2bdt+d^2t^2$$
$$d'(t)=2ac+2c^2t+2bd+2d^2t$$
$$d'(t)=2ca+2c^2t+2db+2d^2t$$
$$0=2ca+2c^2t+2db+2d^2t$$
$$-ca-db=c^2t+d^2t$$
$$\frac{-ca-db}{c^2+d^2}=t$$

After solving t, we can simply put it back to the original equation, then we will get the finally equation:

$$d^2 = (a + c\frac{-ca - db}{c^2 + d^2})^2 + (b + d\frac{-ca - db}{c^2 + d^2})^2 \qquad d = \sqrt{(a + c\frac{-ca - db}{c^2 + d^2})^2 + (b + d\frac{-ca - db}{c^2 + d^2})^2}$$

Method 3: Using cross product and dot product:



$\left|\overrightarrow{PQ} \times \overrightarrow{Pm}\right| = \mathrm{d}\left|\overrightarrow{\mathrm{PQ}}\right| \sin\theta$ where pm is a point perpendicular to the line

Solution: $\dfrac{\left|\overrightarrow{PQ} \times \overrightarrow{Pm}\right|}{\left|\overrightarrow{\mathrm{PQ}}\right| \sin\theta} = \mathrm{d}$

Method 4: Using derivation requires that the line is not vertical or horizontal.

It is possible to produce another expression to find the shortest distance of a point to a line. This derivation also requires that the line is not vertical or horizontal.

The point P is given with coordinates (x_0, y_0). The equation of a line is given by $y = mx + k$. The equation of the normal of that line which passes through the point P is given $y = \dfrac{x_0 - x}{m} + y_0$.

The point at which these two lines intersect is the closest point on the original line to the point P. Hence:

$$mx + k = \frac{x_0 - x}{m} + y_0.$$

We can solve this equation for *x*,

$$x = \frac{x_0 + my_0 - mk}{m^2 + 1}.$$

The y coordinate of the point of intersection can be found by substituting this value of *x* into the equation of the original line,

$$y = m\frac{(x_0 + my_0 - mk)}{m^2 + 1} + k.$$

Using the ~~equation for finding the distance~~ between 2 points, $d = \sqrt{(X_2 - X_1)^2 + (Y_2 - Y_1)^2}$, we can deduce that the formula to find the shortest distance between a line and a point is the following:

$$d = \sqrt{\left(\frac{x_0 + my_0 - mk}{m^2+1} - x_0\right)^2 + \left(m\frac{x_0 + my_0 - mk}{m^2+1} + k - y_0\right)^2}.$$

Recalling that $m = -a/b$ and $k = -c/b$ for the line with equation $ax + by + c = 0$, a little algebraic simplification reduces this to the standard expression.

8- Find the equation of tangent plane to the equation C: $x^2 + y^2 + z^2 = 9$ at point P(2, 2, 1).

We have $f(x,y,z) = x^2 + y^2 + z^2 = 9$

Pre calculus method: Normal to a Sphere at a point is the vector from center of the Sphere to the point.

Calculus Method: we have to find use partial differentiation to find normal vector of the tangent plane.

$\frac{df}{dx}(x,y,z) = 2x$ $\frac{df}{dy}(x,y,z) = 2y$ $\frac{df}{dz}(x,y,z) = 2z$ Then, since we have point P(2,2,1), we can simply find the tangent plane's normal vector by solving the equation.

$\frac{df}{dx}(x,y,z) = 2x = 2*2 = 4$ $\frac{df}{dy}(x,y,z) = 2y$ $2*2 = 4$ $\frac{df}{dz}(x,y,z) = 2z = 2*1 = 2$

Thus, we get the normal vector for the plane, which is $\langle 4,4,2 \rangle$ Thus the equation of the tangent plane to the surface $x^2 + y^2 + z^2 = 9$ at the point

$(2, 2, 1)$ is $4(x-2) + 4(y-2) + 2(z-1) = 0$, 、

that is $4x + 4y + 2z = 18$. Actually we will take the normal to be n = 2i+2j+k. The extra factor 2 is not needed.

SO, it can be simplified to solution: $2x + 2y + z = 18$

9- Find the parametric equation of line of intersection between two tangent planes to the sphere with equation C: $x^2 + y^2 + z^2 = 9$ at point P(1, -2, 2) and Q(1,2,2).

- We know that center of the sphere is located at O(0,0,0)
- Find Normal vectors of the tangent planes are from their tangent point to the center of the sphere(0,0,0):

  $QP = \langle 1,-2,2 \rangle$ $QP = \langle 1,-2,2 \rangle$

- Next we get the equations for the tangent planes at points P and Q:

Plane at P: $x - 2y + 2z = d$

- Find d: where P=(1,-2,2) $d = 9$
- The equation of plane tangent to the sphere at P is $x - 2y + 2z = 5$

Plane at Q: $x + 2y + 2z = d$

- Find d: where P=(1,2,2) $d = 9$
- The equation of plane tangent to the sphere at P is $x + 2y + 2z = 5$

-Next solve for the line of intersection:

Let $z = t$, $\begin{matrix} x - 2y + 2z = 9 \\ x + 2y + 2z = 9 \end{matrix}$

$x = 9 + 2y - 2t$ (Solve x for in equation 1)

$9 + 2y - 2t + 2y + 2t = 9$ (Plug in equation 2)

$9 + 4y = 9$; $y = 0$ (Continue to solve x respect to t)

$x = 9 + 0 - 2t = 9 - 2t$

Finally, we get the parametric equation for the line of intersection: $\begin{matrix} x = 9 - 2t \\ y = 0 \\ z = t \end{matrix}$

10 Show that

A) The three points A (1, 2,-1), B (2,-1, 1) and C (3,-4, 3) are collinear and find the line which passes through them.

$\overrightarrow{AB}$ is parallel to $\overrightarrow{BC}$ if their vectors can be expressed as scalar multiples of each other.

$\overrightarrow{AB}$ is collinear to $\overrightarrow{BC}$ if they are parallel and share a common point.

$\overrightarrow{AB} = \langle 1,-3,2 \rangle$ $\quad \overrightarrow{BC} = \langle 1,-3,2 \rangle$

$\overrightarrow{AB} \parallel \overrightarrow{BC}$

$\overrightarrow{AB}$ and $\overrightarrow{BC}$ share the point B and are collinear.

B) The four points A (1,2,1) , B (0,-1,1) , C(1,-4,2) and D(2,1,0) are not coplanar. Then find the distance of each point to the plane created by the other three points.

Calculate Vectors: $\begin{matrix} \overrightarrow{AB} = \langle 1,3,0 \rangle \\ \overrightarrow{BC} = \langle 1,-3,1 \rangle \\ \overrightarrow{CD} = \langle 1,5,-2 \rangle \\ \overrightarrow{AD} = \langle 1,-1,-1 \rangle \end{matrix}$

Calculate Normals:

$$\overrightarrow{N_{ABC}} = \overrightarrow{AB} \times \overrightarrow{BC} = \begin{vmatrix} i & j & k \\ 1 & 3 & 0 \\ -1 & 3 & 1 \end{vmatrix} = i(3) + j(0) + k(3) + 3k - 0i - j = \langle 3,-1,6 \rangle$$

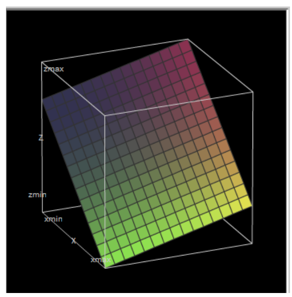
$$\overrightarrow{N_{ABD}} = \overrightarrow{AB} \times \overrightarrow{AD} = \begin{vmatrix} i & j & k \\ 1 & 3 & 0 \\ -1 & 1 & 1 \end{vmatrix} = 3i + 0j + k + 3k - 0i - j = \langle 3,-1,4 \rangle$$

$$\overrightarrow{N_{ACD}} = \overrightarrow{CD} \times \overrightarrow{AD} = \begin{vmatrix} i & j & k \\ -1 & -5 & 2 \\ -1 & 1 & 1 \end{vmatrix} = -5i - 2j - k - 5k - 2i + j = \langle -7,-1,-6 \rangle$$

$$\overrightarrow{N_{BCD}} = \overrightarrow{CD} \times \overrightarrow{BC} = \begin{vmatrix} i & j & k \\ -1 & -5 & 2 \\ -1 & 3 & -1 \end{vmatrix} = 5i - 2j - 3k - 5k - 6i - j = \langle -1,-3,-8 \rangle$$

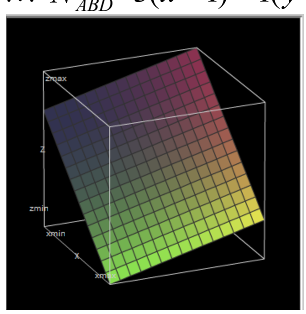
Equations of Planes

Point A $(1,2,1)$ with $\overrightarrow{N_{ABC}}: 3(x-1) - 1(y-2) + 6(z-1) = 0$



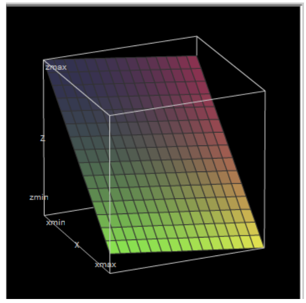
Plane ABC: $3x - y + 6z = 7$

Point A $(1,2,1)$ with $\overrightarrow{N_{ABD}}: 3(x-1) - 1(y-2) + 4(z-1) = 0$



Plane ABD: $3x - y + 4z = 5$

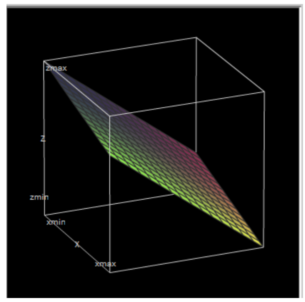
Point A $(1,2,1)$ with $\overrightarrow{N_{ACD}}: -7(x-1) - (y-2) - 6(z-1) = 0$



Plane ACD: $7x + y + 6z = 15$

Point B $(0,-1,1)$ with $\overrightarrow{N_{BCD}} : -1(x) - 3(y+1) - 8(z-1) = 0$

Plane BCD: $x + 3y + 8z = 5$



The shortest distance from point (X,Y,Z) to plane $ax + by + cz + d = 0$ is

$$D = \frac{|aX + bY + cZ + d|}{\sqrt{a^2 + b^2 + c^2}}$$

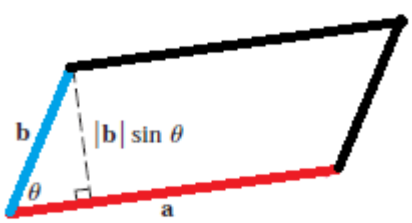
Point A(1,2, 1) & Plane BCD: $$D = \frac{|-1(1) - 3(2) - 8(1) + 7|}{\sqrt{(-1)^2 + (-3)^2 + (-8)^2}} = \frac{8}{\sqrt{74}}$$

Point B (0,-1,1) & Plane ACD: $$D = \frac{|7(0) + 1(-1) + 6(1) - 15|}{\sqrt{(7)^2 + (1)^2 + (6)^2}} = \frac{10}{\sqrt{86}}$$

Point C(1,-4,2) & Plane ABD: $$D = \frac{|3(1) - 1(-4) + 4(2) - 5|}{\sqrt{(3)^2 + (-1)^2 + (4)^2}} = \frac{10}{\sqrt{26}}$$

Point D(2,1,0) & Plane ABC: $$D = \frac{|3(2) - 1(1) + 6(0) - 7|}{\sqrt{(3)^2 + (-1)^2 + (6)^2}} = \frac{2}{\sqrt{46}}$$

11- Find the relation between magnitude Dot product and Cross product.



Area of the parallelogram= $|\vec{a} \times \vec{b}| = (|a| \bullet |b| \sin\theta)$

$$\left|\vec{a}\times\vec{b}\right|^2 = (|a|\cdot[b[\sin\theta)^2 = (|a|\cdot|b|)^2(1-(\cos\theta)^2) = (|a|\cdot|b|)^2 - (|a|\cdot|b|)^2(\cos\theta)^2$$

$$because.........\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|a|\cdot|\vec{b}|}$$

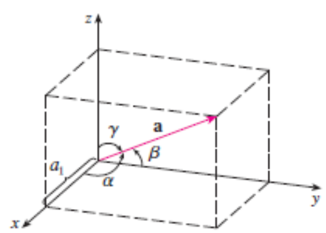
$$\left|\vec{a}\times\vec{b}\right|^2 = \left|\overrightarrow{(a}|\bullet|\vec{b}|\right)^2 - (\left|\vec{a}\times\vec{b}\right|^2)\times(\frac{\vec{a}\bullet\vec{b}}{\left|\overrightarrow{(a}|\bullet|\vec{b}|\right)})^2$$

$$\left|\vec{a}\times\vec{b}\right|^2 = \left|\overrightarrow{(a}|\bullet|\vec{b}|\right)^2 - (\vec{a}\bullet\vec{b})^2$$ take square root for bother sides.

Finally: The relation between Cross product and Dot product is

Solution : $\left|\vec{a}\times\vec{b}\right| = \sqrt{(|\vec{a}|\bullet|\vec{b}|) - (\vec{a}\bullet\vec{b})^2}$

12- Show that sum of the squares of the directional cosines is one.



Point $x(x,0,0)$ $y(0,y,0)$ $z(0,0,z)$ $O(0,0,0)$

- We set vector $\vec{a}$=vector $\overrightarrow{OP}$, where point P is (x,y,z)
- We have to find 4 vectors: $\overrightarrow{OP} = < x-0, y-0, z-0 > = < x, y, z >$

$$\overrightarrow{Nx} = \langle x-0, 0-0, 0-0\rangle = \langle x,0,0\rangle$$

$$\overrightarrow{Ny} = \langle 0-0, y-0, 0-0\rangle = \langle 0,y,0\rangle$$

$$\overrightarrow{Nz} = \langle 0-0, 0-0, z-0\rangle = \langle 0,0,z\rangle$$

Then, we can apply the equation of angle between two vectors:

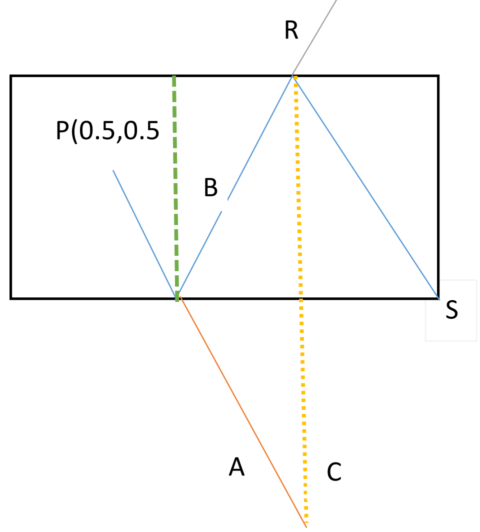
$$Cos\alpha = \frac{\overrightarrow{op}\cdot\overrightarrow{Nx}}{|op||Nx|} = \frac{x^2+0+0}{\sqrt{x^2+y^2+z^2}\cdot\sqrt{x}} = \frac{x}{\sqrt{x^2+y^2+z^2}}$$

$$Cos\beta = \frac{\overrightarrow{op}\cdot\overrightarrow{Ny}}{|op||Ny|} = \frac{0+y^2+0}{\sqrt{x^2+y^2+z^2}\cdot\sqrt{y}} = \frac{y}{\sqrt{x^2+y^2+z^2}}$$

$$Cos\,v = \frac{0+0+z^2}{\sqrt{x^2+y^2+z^2}\cdot\sqrt{z}} = \frac{z}{\sqrt{x^2+y^2+z^2}}$$

$$So, Cos^2\alpha + Cos^2\beta + Cos^2 v = \frac{x^2+y^2+z^2}{(\sqrt{x^2+y^2+z^2})^2} = 1$$

13.) You want to place the ball in to pocket at point S, while it bounces at point Q and R. What are the locations of point Q and R on the rim of the pool table as shown?



X

Use the diagram and mirror image to solve the problem

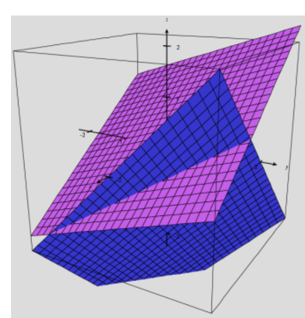
Let Q(x,0)   QA=x-0.5   QA/QC=BA/RC   (x-0.5)/QC=0.5/1   QC=2x-1
QC=CS=2X-1   QC+CS+QA+0.5=2   X=0.8   Q(0.8,0), R(1.4, 1)

14) Find the equation of the plane that passes through the point $P(-1,2,1)$ and contains the line of intersection of the planes $x+y-z=2$ and $2x-y+3z=1$.

We have: point $P(-1,2,1)$, $x+y-z=2$ and $2x-y+3z=1$

So we let $z=t$, we have $x+y=2+t$, $2x-y=1-3t$

Then we have $x=1-2t/3$; $y=1+5t/3$; $z=t$



The directional vector is calculated through the coefficients of t:

$z = t \quad \vec{v} = <-2/3,\ 5/3,\ 1>$

Next we need a point so we set $t = 0$ to get $Q(1,1,0)$ PQ=<2, -1, -1>

$\vec{N} = PQ \times \vec{v} = <2/3,\ -4/3,\ 8/3>$ point $P(-1,2,1)$

2/3(x+1)-4/3(y-2) +8/3(z-1) =0 The plane is x-2y+4z=-1

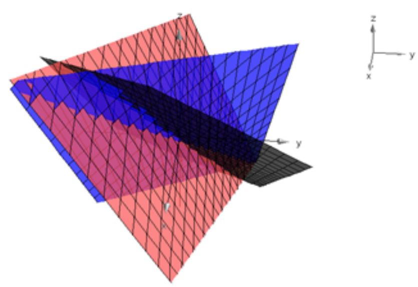
15）. Given the system of equations $\begin{cases} 2x+y-z=2 \\ x+2y+z=1 \\ 2x-y-2z=3 \end{cases}$ (a) Solve for the line of intersection of the first two planes. (b) Solve for the point of intersection of the above line with the third plane.

(a) Starting out with the first two equations $\begin{cases} 2x+y-z=2 \\ x+2y+z=1 \end{cases}$, $\begin{cases} 2x+y=2+t \\ -2x-4y=-2+2t \end{cases}$,

y=t/3

X=4t/3+1, z=t

$$\begin{cases} x = \frac{4}{3}t+1 \\ y = \frac{1}{3}t+0 \\ z = t+0 \end{cases}$$



(b) Plugging in the values

$2(\frac{4}{3}t+1)-(\frac{1}{3}t)-2t=3 \qquad \frac{8}{3}t-\frac{1}{3}t-2t=3-2 \qquad t=3=z \ \begin{cases} x=4 \\ y=1 \\ z=3 \end{cases}$

The point is (4,1,3)

16) Graph $x+y+z=4$. Find the closest point on the above plane to point P(-1,-3,2)

$\vec{n} = <1,\ 1,\ 1>$ Q(0,4,0), $PQ = <1,\ 7,\ -2>$

$$Comp_{\vec{n}}^{PQ} = \frac{\vec{n} \bullet PQ}{|\vec{n}|} = 2\sqrt{3}$$

17). Graph $2x + y - z = 2$ Find the shortest distance from the plane to the origin in 4 different methods.

$\vec{n} = <2,\ 1,\ -1>$ Q(0,0,0), $P = (1,\ 0,\ 0)$

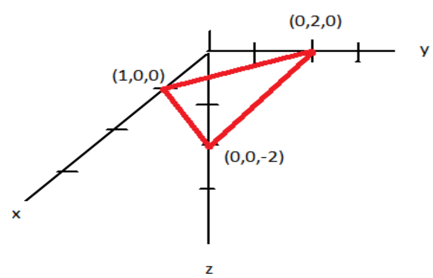
Method 1: use dot product

QP = (-1, 0, 0) $\quad \vec{n} = <2,\ 1,\ -1>$

$$Comp_N^{QP} = \frac{2}{\sqrt{2^2 + 1^2 + (-1)^2}} = \frac{2}{\sqrt{6}} = \frac{\sqrt{6}}{3}$$

Method 2: Use cross Product

$$Dist = \frac{\left|\vec{A} \bullet (\vec{B} \times \vec{C})\right|}{\left|\vec{B} \times \vec{C}\right|}$$



Method 3: Using PARTIAL derivatives

$$\begin{cases} x = 1 + t - s \\ y = 2s \\ z = 2t \end{cases}$$

$$F((t,s) = (1 + t - s)^2 + (2s)^2 + (2t)^2$$

$$\begin{cases} \dfrac{df}{dt} = 2(1 + t - s) + 4t = 0 \\ \dfrac{df}{ds} = 2(1 + t - s)(-1) + 4s = 0 \end{cases} \qquad t = -1/4,\ s = 1/4 \qquad PQ = <1/2,\ 1/2,\ -1/2> \qquad D = \frac{\sqrt{3}}{2}$$

Method 4: Geometry

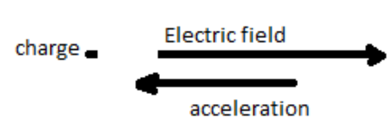
Using the origin and the normal vector, the parameterized form of the perpendicular line is $\vec{r}(t) = <0,\ 0,\ 0> + <2,\ 1,\ -1> t$

$\vec{r}(t) = <2t,\ t,\ -t>$ Plug this in back to the original equation of the plane.

Solve for parameterize $t = \frac{1}{3}$

Plug t into r(t) which equals to $(\frac{2}{3}, \frac{1}{3}, -\frac{1}{3})$

$$= \sqrt{(\tfrac{2}{3})^2 + (\tfrac{1}{3})^2 + (-\tfrac{1}{3})^2} = \sqrt{\tfrac{4}{9} + \tfrac{1}{9} + \tfrac{1}{9}} = \frac{\sqrt{6}}{3}$$



18). Find mass of charged particle in electric field.

$M\vec{a} = q\vec{E}$ or $M = qE/a$ $|a| = \sqrt{(-16)^2 + (-24)^2} * 10^{20}\ m/s^2$

$|a| = 4\sqrt{52} * 10^{20}\ m/s^2$ $m = \frac{(-2C)\left(\sqrt{52}*10^{10}\frac{N}{C}\right)}{4\sqrt{52}*10^{20}\ m/s^2}$ $m = 2 * 10^{-10}\ kg$

19) When a charge particle moves in a magnetic field. The force excreted on the charge particle is $\sum\vec{F} = q(\vec{v}\times\overrightarrow{B})$where "$\vec{v}$" is the velocity of the particle and $\vec{B}$ is the Magnetic field (vector). We know that $\sum\vec{F} = m\vec{a}$ then $m\vec{a} = q(\vec{v}\times\overrightarrow{B})$ can be used to find the mass of a charge particle if the magnetic field, acceleration, velocity and charge of the particle are known are known. Find acceleration of a charged particle with. m=5$\times10^{-10}kg$ and q=-5×10^{-6} C in a field $\vec{B} =< 10{,}20 >$ $Telsa\ and$ velocity $\vec{v} = < 8, -4 > 10^6$m/s. Find the angle between acceleration/ velocity and acceleration/magnetic field.

We have the equation $m\vec{a} = q(\vec{v}\times\overrightarrow{B})$ simplify it to $\vec{a} = q(\vec{v}\times\vec{B})/m$

First let's find $\vec{v}$

We have $\vec{v} =< 8, -4 > 10^6$ simplify it to $< 8\times10^6, -4\times10^6 >$ make it to $< 8\times10^6, -4\times10^6, 0 >$ so we can have it in 3D the zero is in the Z direction. Make $\vec{B}$=$< 10{,}20 >$ to $< 10{,}20{,}0 >$ to make it to Z direction.

$\overrightarrow{(v}\times\vec{B})$= $< 8\times10^6, -4\times10^6, 0 >\times < 10{,}20{,}0 >$ cross product use matrix

$$\begin{bmatrix} & i,\ j,\ k & \\ 8, & -4 & 0 \\ 10, & 20\,, & 0 \end{bmatrix}10^6 = \begin{bmatrix} -4\times10^6 & 0 \\ 20 & 0 \end{bmatrix}i-\begin{bmatrix} 8\times10^6 & 0 \\ 10 & 0 \end{bmatrix}j+\begin{bmatrix} 8\times10^6 & -4\times10^6 \\ 10 & 20 \end{bmatrix}k$$

$$=0i-0j+2\times10^8 = \langle 0,0,\ 2\times10^8\rangle$$

$\overrightarrow{(v}\times\vec{B})$= $\langle 0,0,\ 2\times10^8\rangle$

$$\frac{q}{m} = \frac{-5\times10^{-6}}{5\times10^{-10}} = -10{,}000$$

$$\vec{a} = \frac{q\left(\vec{v}\times\vec{B}\right)}{m} = -10{,}000 < 0{,}0{,}2\times10^8 >= < 0{,}0{,}-2\times10^{12} >$$

Solution: $\vec{a} =< 0{,}0{,}-2\times10^{12} >$

Finding the angle between the acceleration/velocity

$\vec{a} =< 0{,}0{,}-2\times10^{12} >$ $\vec{v} =< 8\times10^6, -4\times10^6, 0 >$

The equation for finding angle $\theta = \cos^{-1}\frac{\vec{A}\cdot\vec{B}}{|A||B|}$ let A be acceleration vector and B be the Velocity vector

$|\vec{a}|$=$\sqrt{0^2 + 0^2 + (-2\times10^{12})^\wedge 2}$=$\sqrt{4\times10^{24}}$

$|\vec{v}|$=$\sqrt{(8\times10^6)^2 + (-4\times10^6)^2 + 0^2}$=$\sqrt{8\times10^{13}}$

$\vec{A}\cdot\vec{B}$=$a_1b_1 + a_2b_2 + a_3b_3$

$\vec{a}\cdot\vec{v}$=(0)(8×10^6)+(0)($-$ 4×10^6)+(-2×10^{12})(0)=0

So plug it into the angle equation to get $\theta = \cos^{-1}\frac{0}{|\sqrt{8\times10^{13}}\sqrt{4\times10^{24}}|}$=$\cos^{-1} 0$

Solution:$\theta = 90°$

Finding the angle between acceleration and magnetic field
$\vec{B}$=$< 10,20,0 >$ $\vec{a}$ $=< 0,0,-2\times10^{12} >$

Equation for angle is: $\theta = \cos^{-1}\frac{\vec{A}\cdot\vec{B}}{|A||B|}$

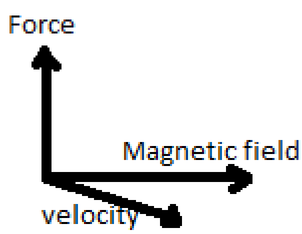
$|\vec{a}|$=$\sqrt{0^2 + 0^2 + (-2\times10^{12})^\wedge 2}$=$\sqrt{4\times10^{24}}$

$|\vec{B}|$=$\sqrt{10^2 + 20^2 + 0^\wedge 2}$ =$\sqrt{500}$

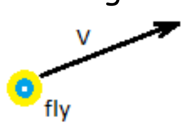
Dot product of $|\vec{a}|\cdot|\vec{B}|$ is =((0)(10)+(0)(20)+(0)(-2×10^{12}))=0

So plug it into the angle equation to get $\theta = \cos^{-1}\frac{0}{|\sqrt{500}\sqrt{4\times10^{24}}|}$=$\cos^{-1} 0$

Solution:$\theta = 90°$



20)When a particle moves in straight line, and then its kinetic energy is KE=1/2m$\vec{v}\cdot\vec{v}$. What is KE (in Joules of a 2miligram fly that is going with velocity $\vec{v}$=<4,3>m/s. We are given velocity and mass so we plug it into the equation.



First we dot product the velocity $\vec{v}\cdot\vec{v} = 25$
The equation is KE=$\frac{1}{2}m\vec{v}\cdot\vec{v}$
M=2mg change it to kg and get 2×10^{-6}
Multiply by ½ and get 1×10^{-6}

So KE= $\times 10^{-6}(25) = 25\times 10^{-6}$
Solution:KE=25×10^{-6}J
1 Joule=1Kg·m^2/s^2

21)-Potential energy of an object at height H is $PE = m\vec{g} * \Delta\vec{H}$. where" g "is acceleration of gravity and $\Delta\vec{H}$ is change in height. A 2kg mass moved from point $A(3,-4)$ to point $B(5,12)$. How much work was done on the object? $\vec{g} =< 0,-10 >$ m/s^2

we find the distance between the two points and we use

$A(3,-4)$ to point $B(5,12)$ which makes the $\Delta\vec{H} =< 2,16 >$

Now we use the equation for potential energy

$PE = m\vec{g} * \Delta\vec{H}$

$W = 2kg < 0,-10 >< 2,16 >$

Now by finding the values we find the work as

$W = 2kg(-160)N$

$W = -320J$

So we are able to use the work formula where F=ma and work is equal to W

The work done is equal to Solution: 320J

22) Central petal force on an object is a fictitious force do to rotation. If a mass moves around a circle of radius R with linear speed $\vec{v}$, then the magnitude of the central petal force will be $|\vec{F}| = m\frac{\vec{v}\cdot\vec{v}}{R}$and the direction is toward the center of the circle. What is acceleration (direction and magnitude) of a mass going around a cirlce (R=0.5m) and $\vec{v} =< -5,12 > m/s$. Show that acceleration is inversely proportional to the radius.

$|\vec{F}| = m\frac{\vec{v}\cdot\vec{v}}{R}$ we have this equation and know that $\sum\vec{F} = m\vec{a}$ so $|m\vec{a}| = m\frac{\vec{v}\cdot\vec{v}}{R}$ so the masses cancel out and the simplified equation becomes $|\vec{a}| = \frac{\vec{v}\cdot\vec{v}}{R}$. This equation can find the magnitude of acceleration.
Dot product for vectors is equal to $\vec{A}\cdot\vec{B}$=$a_1b_1 + a_2b_2 + a_3b_3$
So the dot product for velocity is $\vec{v}\cdot\vec{v}$=$< -5,12 >\cdot< -5,12 >= 169$

The radius is 0 .5m.

$|\vec{a}| = \frac{\vec{v}\cdot\vec{v}}{R} = \frac{169}{.5} = 338$

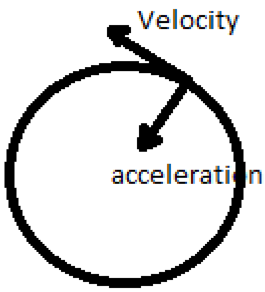
Magnitude is Solution: $|\vec{a}| = 338$

Now we find the direction. We know from the problem that the direction is toward the center of the circle. We also know from the picture that the velocity is perpendicular to the acceleration. So the direction of the acceleration will be the opposite of the velocity to make it perpendicular. It is like two lines that are perpendicular to each other and we know if they are perpendicular to each other then the slopes are negative reciprocal of each other. With vectors we flip them and make one negative.

$\vec{v}$=<-5,12>so the negative flip will give us the direction for acceleration so

Solution:

$\vec{a} =< 12,5 >$ is direction



23) Angular momentum of an object is $\vec{L} = \vec{r}\times\vec{P}$ where change in linear momentum for constant mass is $\Delta\vec{P} = m\Delta\vec{v}$. Find $\vec{L}$ for 0.5 kg object going around a circle

$\vec{r} =< 3,2 > m$ and $\vec{v} =< -10,6 > m/s$

$$\Delta\vec{P} = m\Delta\vec{v}$$

$$\int \Delta\vec{P} = m\int \vec{v}$$

$$\vec{p} = m\vec{v}$$

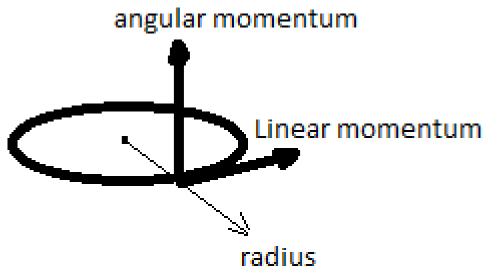
$m\vec{v} = 0.5 < -10,6 > =< -5,3 >$ so $\vec{p} =< -5,3 >$

$\vec{L} = \vec{r}\times\vec{P} \rightarrow$ <3,2> × <-5,3>

$$\begin{matrix} i & j & k \\ 3 & 2 & 0 \\ -5 & 3 & 0 \end{matrix} = \begin{bmatrix} 2 & 0 \\ 3 & 0 \end{bmatrix} i + \begin{bmatrix} 3 & 0 \\ -5 & 0 \end{bmatrix} j + \begin{bmatrix} 3 & 2 \\ -5 & 3 \end{bmatrix} k$$

=(2×0-3×0)i+(0-0)j+(9-(-10)k=0i+0j+19k=$< 0.0,19 >$

$$\vec{L} =< 0.0,19 >$$



24) Angular momentum in 3D: Find $\vec{L}$ for 0.5 kg object going around a circle

$\vec{r} =< 3,4,-2 > m$ and $\vec{v} =< 8,4,6 > m/s$

$\Delta\vec{P} = m\Delta\vec{v}$

$\int \Delta\vec{P} = m\int \vec{v}$

$\vec{p} = m\vec{v}$

$m\vec{v}$=1/2$< 8,4,6 >$so $\vec{p} =< 4,2,3 >$

$\vec{L} = \vec{r}\times\vec{P} \qquad < 3,4,-2 > \times < 4,2,3 >$

$\begin{matrix} i & j & k \\ 3 & 4 & -2 \\ 4 & 2 & 3 \end{matrix}$ = $\begin{bmatrix} 4 & -3 \\ 2 & 3 \end{bmatrix}$i+$\begin{bmatrix} 3 & -2 \\ 4 & 3 \end{bmatrix}$j+$\begin{bmatrix} 3 & 4 \\ 4 & 2 \end{bmatrix}$k

=(12-(-4))i+(9-(-8))j+(6-16)k=16i+17-10k $=< 16,17,-10 >$

$$\vec{L} =< 16,17,-10 >$$

25) Find acceleration of a charged particle with. $m = 5\times10^{-10}kg$ and $q = -5\times10^{-6}$ C in a field $\vec{B} =< 10,20,5 > Telsa\ and$ velocity $\vec{v} =< 8,-4,12 > 10^6 m/s$. Find the angle between acceleration/ velocity and acceleration/magnetic field.

We have the equation $m\vec{a} = q(\overrightarrow{\vec{v}\times B})$ simplify it to $\vec{a} = q(\vec{v}\times\vec{B})/m$

First let's find $\vec{v}$

We have $\vec{v} =< 8,-4,12 > 10^6 m/s$ simplify it to $< 8\times10^6, -4\times10^6, 12\times10^6 >$

$\vec{B}$=$< 10,20,5 > Tesla$

$(\overrightarrow{v}\times\vec{B})$= $< 8\times10^6, -4\times10^6, 12\times10^6 > \times < 10,20,5 >$ cross product use matrix

$$\begin{vmatrix} i & j & k \\ 8\times10^6 & -4\times10^6 & 12\times10^6 \\ 10 & 20 & 5 \end{vmatrix} =$$

$$\begin{bmatrix} -4\times10^6 & 12\times10^6 \\ 20 & 5 \end{bmatrix} i - \begin{bmatrix} 8\times10^6 & 12\times10^6 \\ 10 & 5 \end{bmatrix} j + \begin{bmatrix} 8\times10^6 & -4\times10^6 \\ 10 & 20 \end{bmatrix} k$$

$$= -26\times10^7 i - 8\times10^7 j + 2\times10^8 k = < -26\times10^7, -8\times10^7, 2\times10^8 >$$

$(\overrightarrow{v}\times\vec{B}) = < -26\times10^7, -8\times10^7, 2\times10^8 >$

$$\frac{q}{m} = \frac{-5\times10^{-6}}{5\times10^{-10}} = -10{,}000 C/Kg$$

$\vec{a} = \frac{q(\vec{v}\times\vec{B})}{m} = -10{,}000 < -26\times10^7, -8\times10^7, 2\times10^8 >$

Solution:

$\vec{a} = < 26\times10^{11}, 8\times10^{11}, -2\times10^{12} >$

Finding the angle between the acceleration/velocity

$\vec{a} = < 26\times10^{11}, -8\times10^{11}, -2\times10^{12} >$ $\vec{v} = < 8\times10^6, -4\times10^6, 12\times10^6 >$

The equation for finding angle $\theta = \cos^{-1}\frac{\vec{A}\cdot\vec{B}}{|A||B|}$ let A be acceleration vector and B be the Velocity vector

$\vec{A}\cdot\vec{B} = a_1b_1 + a_2b_2 + a_3b_3$

$$\vec{a}\cdot\vec{v} = 0$$

So plug it into the angle equation to get $\theta = \cos^{-1}\frac{0}{|\ |} = 90\ degree$

Solution: $\theta = 90°$

Finding the angle between acceleration and magnetic field

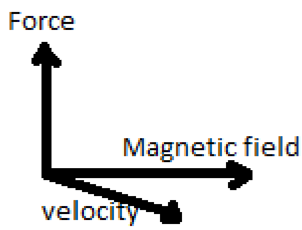
$\vec{B} = < 10,20,5 >$ $\vec{a} = < 26\times10^{11}, 8\times10^{11}, -2\times10^{12} >$

Equation for angle is: $\theta = \cos^{-1}\frac{\vec{A}\cdot\vec{B}}{|A||B|}$

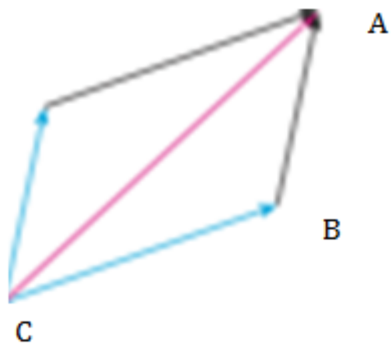
Dot product of $|\vec{a}|\cdot|\vec{B}| = 0$

So plug it into the angle equation to get $\theta = \cos^{-1}\frac{0}{|\ |} = 90\ degree$

Solution: $\theta = 90°$



26)- Find area of a triangle with vertices $A(2,3,6)$ and $B(3,2,5)$ and $C(1,3,4)$.



By finding $\overrightarrow{AB}x\overrightarrow{AC}$ you are finding the area of a parallelogram and the triangle is $\frac{1}{2}$ of the area

$$\overrightarrow{AB}x\overrightarrow{AC} = \begin{matrix} \vec{\imath} & \vec{\jmath} & \vec{k} \\ 1 & -1 & -1 \\ -1 & 0 & -2 \end{matrix} = (2+0)\vec{\imath} + (1+2)\vec{\jmath} + (0-1)\vec{k}$$
$$= 2\vec{\imath} + 3\vec{\jmath} + -1\vec{k}$$

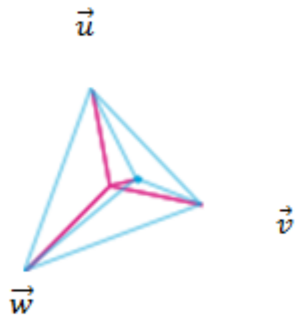
So the area of the triangle is

Solution:

$$\frac{\overrightarrow{AB}x\overrightarrow{AC}}{2} = \frac{\sqrt{4+9+1}}{2} = \frac{\sqrt{15}}{2}$$

27- Find volume of a tetrahedron created by three vectors below

$\vec{v} =< 3,2,5 >$, $\vec{u} =< 3,2,6 >$, $\vec{w} =< 1,4,6 >$



$V = \frac{1}{6}[\vec{u}, \vec{v}, \vec{w}] = \begin{vmatrix} 3 & 2 & 5 \\ 3 & 2 & 6 \\ 1 & 4 & 6 \end{vmatrix}$ $V = \frac{1}{6}[36 + 12 + 60 - 10 - 72 - 36]$, $V = \frac{1}{6}(-10)$

Solution:

$V = \frac{10}{6}$ *because value must be absolute value*

28- Find the net force for $\vec{F}_1 = < -3,5,-6 > N$,$\vec{F}_2 = < 4,2,7 > N$,$\vec{F}_3 = < 8,4,4 >$,$\vec{F}_4 = < -5,1,2 > N$

Then find the acceleration of a 2kg object which influences by these forces.

First we find the components.

$X = -3 + 4 + 8 + -5 = 4$

$Y = 5 + 2 + 4 + 1 = 12$

$Z = -6 + 7 + 4 + 2 = 7$

So now we have $Fnet$ which is $\vec{F}net = < 4,12,7 > N$

Solution:

$F = \sqrt{4^2 + 12^2 + 7^2} = 14.4586$

29.

Dot Product Properties ,So the vector values are shown below

$\vec{v} = \begin{matrix} v_1 \\ v_2 \\ \dots \\ v_n \end{matrix}$ $\vec{w} = \begin{matrix} w_1 \\ w_2 \\ \dots \\ w_n \end{matrix}$

Then by multiplying we get the bottom values

$\vec{v} * \vec{w} = v_1w_1 + v_2w_2 + \cdots. +v_nw_n$

$\vec{w} * \vec{v} = w_1v_1 + w_2v_2 + \cdots. +w_nv_n$

By Commutative property. $v_1w_1 = w_1v_1$ So $\vec{v}\times\vec{w} = \vec{w}\times\vec{v}$

now we bring in a third value $\vec{z}=\begin{bmatrix} z_1 \\ z_2 \\ \dots \\ z_n \end{bmatrix}$

This is the dot product property so we address the property

$(\vec{v} + \vec{w}) * \vec{z} = (\vec{v}\vec{z} + \vec{z}\vec{w})$

$\vec{v} + \vec{w} = \begin{bmatrix} v_1 + w_1 \\ v_2 + w_2 \\ \dots \\ v_n + w_n \end{bmatrix}$ which is true then we dot this with the values of $\vec{z}$

$\begin{bmatrix} z_1 \\ z_2 \\ \dots \\ z_n \end{bmatrix} * \begin{bmatrix} v_1 + w_1 \\ v_2 + w_2 \\ \dots \\ v_n + w_n \end{bmatrix}$ which makes the values

$(w_1+v_1)z_1 + (w_2+v_2)z_2 + \cdots + (w_n+v_n)z_n$which is equal to $(\vec{v} + \vec{w}) * \vec{z}$

now we take the values of the components separately

$\vec{v} * \vec{z} = v_1z_1 + v_2z_2 + \cdots. +v_nz_n$

$\vec{w} * \vec{z} = w_1z_1 + w_2z_2 + \cdots. +w_nz_n$

when we add them together

$\vec{v} * \vec{z} + \vec{w} * \vec{z}=(v_1z_1 + w_1z_1) + (v_2z_2 + w_2z_2) + \cdots. +(v_nz_n + w_nz_n)$

then we factor and this becomes

$(w_1+v_1)z_1 + (w_2+v_2)z_2 + \cdots + (w_n+v_n)z_n$

which is equal to

$(\vec{v} + \vec{w}) * \vec{z} = \vec{v} * \vec{z} + \vec{w} * \vec{z}$

and it shows that it can be applied to vectors

Cross Product.

$\vec{u} =< u_x, u_y, u_z >$
$\vec{v} =< v_x, v_y, v_z >$

Dot $\vec{u} * \vec{v} = < u_x, u_y, u_z > * < v_x, v_y, v_z > = u_x v_x + u_y v_y + u_z v_z$

$|\vec{u}||\vec{v}| \cos\theta = \vec{u} * \vec{v}$

$\cos\theta = \frac{\vec{u} * \vec{v}}{|\vec{u}||\vec{v}|}$

30-

a) Find the line of intersection of

$2x + y - z = 2$ and $x + 2y + z = 1$

a) $2x + y - z = 2$ and $x + 2y + z = 1$

set $z = t$

$\begin{cases} 2x + y - t = 2 \\ x + 2y + t = 1 \end{cases}$

$2x = 2 + t - y, x = \frac{2+t-y}{2}$

substitute x

$\frac{2 + t - y}{2} + 2y - t = 1$

$2 + t - y + 2y - t = 2$

$3y = t \quad y = \frac{t}{3}$

so we can substitute in y $x = \frac{2+t-\frac{t}{3}}{2} = 1 + \frac{t}{2} - \frac{t}{6} = 1 + \frac{3t}{6} - \frac{t}{6} = 1 + \frac{2t}{6} = 1 + \frac{t}{3}$

Solution:

$$\begin{cases} x = 1 + \frac{t}{3} \\ y = \frac{t}{3} \\ z = t \end{cases}$$

b) Find the acute angle of the above planes.

use normal vector of the plane

$s_1: \vec{N}_1 = < 2, 1, -1 >, s_2: \vec{N}_2 = < 1, 2, 1 >$

$$\cos\theta = \frac{\vec{N}_1 * \vec{N}_2}{|\vec{N}_1||\vec{N}_2|} = \frac{2*1+1*2+(-1)*1}{\sqrt{2^2+1^2+1^2}*\sqrt{1^2+2^2+1^2}} = \frac{2+2-1}{\sqrt{6}*\sqrt{6}} = \frac{3}{6} = \frac{1}{2}$$

Solution:

$$\theta = \cos^{-1}(\frac{1}{2}) = \frac{\pi}{3}\ or\ 60°$$

c) Find point of intersection of the above planes with plane $x - y - z = 3$.

Line equation is $\begin{cases} x = 1 + \frac{t}{3} \\ y = \frac{t}{3} \\ z = t \end{cases}$ Plane is $x - y - z = 3$

Put in the values into the plane equation

$$\left(1 + \frac{t}{3}\right) - \frac{t}{3} - t = 3$$

$1 - t = 3, \quad t = -2s$ When $t = 2s$

$$\begin{cases} x = 1 + \frac{-2}{3} = \frac{1}{3} \\ y = \frac{-2}{3} \\ z = -2 \end{cases}$$

Solution: point of intersection $(x, y, z) = (\frac{1}{3}, \frac{-2}{3}, -2)$

31.) Find the closest point on the plane $\mathrm{x + y + z = 4}$ to the origin.

The normal vector of the plane is $< 1,1,1 >$, and the origin is at (0, 0, 0),

Therefore, $r(t) ==< 0,0,0 > + \frac{4}{3} < 1,1,1 >$. In parametric form, we have

$$x = t$$

$$y = t$$

$$z = t$$

Given the equations of x, y, and z, we can now find t; plug the equations into the plane function.

$$t + t + t = 4$$

$3t = 4 \quad t = \frac{4}{3}$

Now that we found the value of t, we can plug it into r(t) to obtain the point on the plane that is closest to the origin.

$P =< 0,0,0 > + \frac{4}{3} < 1,1,1 > \quad P = \left(\frac{4}{3}, \frac{4}{3}, \frac{4}{3}\right)$

32.) Find the line of intersection of $2x + y - z = 2$ and $x + 2y + z = 1$ then find the acute angle of the plane

Substitute a variable with zero in order to find the values.

Let $x = 0$:
$y - z = 2$

$2y + z = 1$

Solve for z.

$y = z + 2$

$2(z + 2) + z = 1$

$2z + 4 + z = 1$

$3z = -3 \quad z = -1 \quad y = 1$

The set of values we obtain is (0, 1, -1). Now we can find the equations for the line of intersection. But first we must find the vector by using the cross product.

$$< 1,2,1, > * < 1,2,1 > = \begin{vmatrix} i & j & k \\ 2 & 1 & -1 \\ 1 & 2 & 1 \end{vmatrix}$$

We find that this equates to (3, -3, 3). Now we can plug it into r(t).

$$r(t) == < 0,1,-1 > + t < 3,-3,3 >$$

In parametric form:

$x = 3t$

$y = 1 - 3t$

$z = -1 + 3t$

To find the angle between the planes, we use the formula $\cos(\theta) = \frac{a_1 b_1 + a_2 b_2 + a_3 b_3}{|A||B|}$

So we get: $\theta = \cos^{-1}\frac{(2+2-1)}{\sqrt{1+4+1}\,\sqrt{4+1+1}} = \cos^{-1}\frac{3}{\sqrt{6}\,\sqrt{6}} = \cos^{-1}\frac{1}{2} = \pi/3$

33.) A particle at $t = 0$ started at point (4, 0, 3) in the direction $v =< -1, 2, 0 >$ bounces off the surface $x - y + z = 4$.

a.) Find the location of the particle after 4 sec.

To find the point after 4 seconds, we must obtain the parametric equations. Doing so, we will use $r(t) =< x, y, z > +t < a, b, c >$.

Thus our system of parametrics is

$x = 4 - \frac{1}{\sqrt{5}}t$

$y = \frac{2}{\sqrt{5}}t$

$z = 3$

Now we want to find out at what time the particle reaches the plane, so we plug the parametrics into the plane function:

$4 - \frac{1}{\sqrt{5}}t - \frac{2}{\sqrt{5}}t + 3 = 4 \qquad -\frac{3}{\sqrt{5}}t = -3 \qquad t = \sqrt{5}\,Sec$

Now we know that the particle reaches the plane after one second. Now we must find where the particle is at after it bounces off the plane, three seconds later:

$t = 4 - \sqrt{5};$

$x = \sqrt{5}$

$y = 2(4 - \sqrt{5}) = 8 - 2\sqrt{5}$

$z = 3$

The particle is at $(\sqrt{5}, 8 - 2\sqrt{5}, 3)$ after four seconds.

b.) Find the displacement during the interval [0, 4] sec.

Use distance formula

34.) A particle at $t = 0$ was detected at point (2, 1, 3). Then after it bounced off the surface $x + y + z = 3$ it was observed at point (2, 2, 2). For how long was the particle missing?

$$\vec{N} = < 1,1,1 > = \vec{v}$$

$$\frac{\vec{v}}{|\vec{v}|} = \frac{1,1,1}{\sqrt{3}} = <\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}>$$

$$\text{Line} \begin{cases} x = 2 + \frac{1}{\sqrt{3}}t \\ y = 2 + \frac{1}{\sqrt{3}}t \\ z = 2 + \frac{1}{\sqrt{3}}t \end{cases} \qquad t = -\sqrt{3}$$

Because of symmetry, the time it takes is $-\sqrt{3} \cdot 2 = -2\sqrt{3}$.

$$Q = \begin{cases} x = 2 + \frac{1}{\sqrt{3}}(-2\sqrt{3}) = 0 \\ y = 0 \\ z = 0 \end{cases} \qquad Q' = (0,0,0)$$

Find PQ': $< -2, -1, -3 >$, $PQ' = \vec{v}$

Normative: $\frac{\vec{v}}{|\vec{v}|} = <\frac{-2,-1,-3}{\sqrt{14}}> = <\frac{-2}{\sqrt{14}}, \frac{-1}{\sqrt{14}}, \frac{-3}{\sqrt{14}}>$

Find the line equation: $PQ' = \begin{cases} x = 2 - \frac{2}{\sqrt{14}} \\ y = 1 - \frac{1}{\sqrt{14}} \\ z = 3 - \frac{3}{\sqrt{14}} \end{cases}$

Solve for t: $2 - \frac{2}{\sqrt{14}}t + 1 - \frac{1}{\sqrt{14}}t + 3 - \frac{3}{\sqrt{14}}t = 3$

$6 - \frac{6}{\sqrt{14}}t = 3 \qquad t = \frac{\sqrt{14}}{2}$ Total time: $2 \cdot \frac{\sqrt{14}}{2} = \sqrt{14}\, s$

35.) Given $(t) =< t, ln|sect|, 2 >$, $0 \le t \le ln2$.

a.) Show that $r(t)$ is a smooth vector function.

A smooth vector function is continuous on the interval and its derivative cannot be zero. We must differentiate $r(t)$ to make sure it is continuous and not equal to zero.

$$r'(t) = < x'(t), y'(t), z'(t) >$$

$\frac{dx}{dt} = 1 \qquad \frac{dy}{dt} = \tan(t) \qquad \frac{dz}{dt} = 0$

Since we are working with just two dimensions, z may be equal to zero. Plugging in 0 and ln2, r(t) is continuous and r'(t) is not equal to zero. Therefore, it is a smooth vector function.

b.) Find the length of the curve from t = 0 to t = ln2.

To find the length, we must use the arc length formula.

$$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}\, dt$$

Since we know dx/dt, dy/dt, and dz/dt,

$$L = \int_0^{ln2} \sqrt{(1)^2 + (\tan(t))^2 + (0)^2}\, dt = \int_0^{ln2} \sqrt{sec^2 t}\, dt$$

$$L = \int_0^{ln2} \sec(t)\, dt = [\ln|\sec(t) + \tan(t)|]_0^{ln2}$$

$$= \ln\, |\sec(ln2) + \tan\,(ln2)| - \ln\, |\sec(0) + \tan\,(0)|$$

$$= \ln\, |\sec(ln2) + \tan\,(ln2)| - \ln\, |1|$$

$$L = \ln\, |\sec(ln2) + \tan\,(ln2)|$$

36.) A particle with initial velocity $v = 2\sqrt{2}\, m/s$ and $\theta = \frac{\pi}{4}$ is launched from the origin in a field with $a = < 0, -2, 0 > m/s^2$. Find all the points that the particle intersects the plane S: $x + y + z = 3$.

Let Q' be a point equidistant from Q, the final position of the particle, to the plane. Let P be the initial point of the particle.

37-Two planes $S_1 : x + 2y + z = 4$ *and* $S_2 : x - y + z = 1$ intersect at line L.

Find the distance of the point P (1, 0, 2) to both planes and line L.

Set $z = t$ solve equation

a) We get $x = 4 - t - 2y$ from plane 1, then plug into plane 2

$4 - t - 2y - y + y = 1$ we get $y = 1, \rightarrow\rightarrow\rightarrow x = 4 - t - 2 = 2 - t$

Solution: Line intersection: $\begin{cases} x(t) = 2 - t \\ y(t) = 1 \\ Z(t) = t \end{cases}$

b) Distance from S_1 to P (1,0,1), find one point on the plane, set x and y equal to 0

$A(0,2,0), \overrightarrow{AP} = \langle 1,-2,2\rangle, Normal\vec{N} = \langle 1,2,1\rangle$

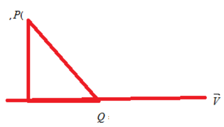
Solution:Distance= $Comp\,\frac{\overrightarrow{AP}}{\vec{N}} = \frac{\overrightarrow{AP}\bullet\vec{N}}{\left|\overrightarrow{AP}\bullet\vec{N}\right|} = \frac{\langle 1,-2,2\rangle\langle 1,2,1\rangle}{\sqrt{1+4+1}} = \left|\frac{-\sqrt{6}}{6}\right| = \frac{\sqrt{6}}{6}$

c) Distance from S_2 to P (1,0,1), find one point on the plane set z,y equal to 0

$B(1,0,0), \overrightarrow{BP} = \langle 0,0,2\rangle, Normal\vec{N} = \langle 1,-,1\rangle$

Solution:Distance= $Comp\,\frac{\overrightarrow{BP}}{\vec{N}} = \frac{\overrightarrow{BP}\bullet\vec{N}}{\left|\overrightarrow{BP}\bullet\vec{N}\right|} = \frac{\langle 1,0,0\rangle\langle 1,-1,1\rangle}{\sqrt{1+1+1}} = \left|\frac{1}{\sqrt{3}}\right| = \frac{\sqrt{3}}{3}$

d) $\begin{cases} x(t) = 2-t \\ y(t) = 1 \\ Z(t) = t \end{cases}$



one point on the line, which is $Q = (2,1,0)$, $P(1,0,1), \overrightarrow{PQ} = \langle 1,1,-2\rangle$ $\vec{V} = \langle 1,0,1\rangle$

Distance from P to Q:

$\left|\overrightarrow{PQ}\right|) = \sqrt{1+1+4} = \sqrt{6}, \left(Comp\,\frac{\overrightarrow{PQ}}{\vec{V}}\right) = \frac{\langle 1,1,-2\rangle\bullet\langle -1,0,1\rangle}{\left|\sqrt{1+0+1}\right|} = \frac{3}{\sqrt{2}}$

Solution:

$$D = \sqrt{(\left|\overrightarrow{PQ}\right|)^2 - \left(Comp\,\frac{\overrightarrow{PQ}}{\vec{V}}\right)^2} = \sqrt{(\sqrt{6})^2 - (\frac{3}{\sqrt{2}})^2} = \frac{\sqrt{6}}{2}$$

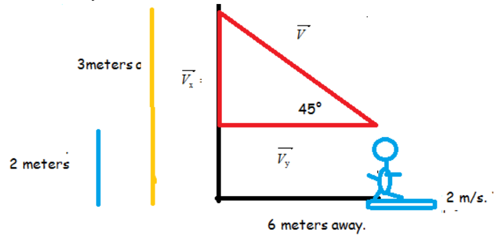
38)- What is the mirror image of vector $V\langle 1,2,1\rangle$ respect to $U\langle 2,-1,2\rangle$

$\vec{v}* = 2proj_{\vec{n}}^{\vec{v}} - \vec{v}$ Therefore, we can use this formula to solve $\vec{v}*$

Solution:

$$\vec{V} = 2\frac{\vec{V}\cdot\vec{U}}{\left|\vec{U}\right|} - \vec{V} = 2\cdot\frac{2-2+2}{\sqrt{4+1+4}}\cdot\frac{\langle 2,-1,2\rangle}{\sqrt{4+1+4}} - \langle 1,2,1\rangle = \frac{\langle 8,-4,8\rangle}{9} - \langle 1,2,1\rangle = \left\langle\frac{-1}{9}, \frac{-22}{9}, \frac{-1}{9}\right\rangle$$

39- A boy on a skateboard is moving toward a basketball hoop at 2 m/s. The hoop is 3meters above the ground. At time t = 0, the boy throws the ball from 2 meters above the ground at an angle of 45° when he is 6 meters away. How fast (in respect to him) does he have to throw the ball in order to make a basket?



40- Find the equation of the plane that passes through the point (-1, 2, 1) and contains the line of intersection of the planes $x+y-z=2$ and $2x-y+3z=1$

- Set $z=t$ solve the system: $x+y-z=2$ and $2x-y+3z=1$
- solve the x respect to t

$$4+2t-2y-y+3t=1 \rightarrow\rightarrow y=1+\frac{5}{3}t$$

$$x=2+t-u=2+t-\left(\frac{3-2t}{3}\right)\rightarrow\rightarrow\rightarrow x=1-\frac{2}{3}t$$

Line eqaiton $\begin{cases} x=1-\frac{2}{3}t \\ y=1+\frac{5}{3}t: \\ z=t \end{cases}$

Select Point Q (1,1,0) P (-1,2,1) SO $\overrightarrow{PQ}=(2,-1,-1)$

Directional vector of line equation is $\overline{V}\left\langle -\frac{2}{3},\frac{5}{3},1\right\rangle$

Find normal vector of plane:

$$\overline{N}=\overline{PQ}\times\overline{V}=\begin{vmatrix} i & j & k \\ 2 & -1 & -1 \\ \frac{-2}{3} & \frac{5}{3} & 1 \end{vmatrix}=\frac{2}{3}i-\frac{4}{3}j+\frac{8}{3}k$$ then find the equation of the plane:

Solution: $x-2y+4z=-1$

41- Find the work done to move a 4kg mass with acceleration $\vec{a}=<2,-3,1>m/s^2$ Form point $P(2,4,5)$ to point $Q(1,-3,2)$.

Solution: $W = \vec{F} \bullet \vec{d} = m\vec{a} \bullet (PQ) = 4 < 2,-3,1 > \bullet < -1,-7,-3 >= 64\ \ Joules$

42- Find magnitude of torque if the moment arm is $\vec{r}(t) =< 2,2,-1 >$and applied force is $\vec{F} =< 3,4,-12 >$. What is the angle between moment arm and the force in above question?

$$|T| = |\vec{r} \times \vec{F}| = \begin{vmatrix} i & j & k \\ 2 & 2 & -1 \\ 3 & 4 & -12 \end{vmatrix} = |-20i + 27j + 2k| = \sqrt{1133}$$

$$T = |r||F|\sin\theta \quad |r| = \sqrt{4+4+1} = 3, |F| = \sqrt{9+16+14} = 13 \quad \sin\theta = \frac{\sqrt{1133}}{3 \bullet 13}$$

Solution: $\theta = \sin^{-1}(\frac{\sqrt{1133}}{39})$

43 a) Find a parametric equation of a line L whose passes through point A(1, -1, 2), B(2, -2, 3)

$\overrightarrow{AB} = 2 - 1, -2 + 1, 3 - 2$

= (1, -1, 1)

$$\overline{AB} = \frac{X - A_x}{AB_x}, \frac{Y - A_y}{AB_y}, \frac{Z - A_z}{AB_z}$$

$$\overline{AB} = \frac{X-1}{1} = \frac{Y+1}{-1} = \frac{Z-2}{1} \rightarrow \boxed{\mathrm{X = 1 + t, Y = -1 - t, Z = 2 + t}}$$

43 b) Find an equation of the plane S_1 whose passes through points C(3,-2,2), D(3,2,4), E(4,2,1)

$\overrightarrow{CD}$ $(3 - 3, 2 + 2, 4 - 2)$

$=< 0, 4, 2 >$

$\overrightarrow{BC}$ $(4 - 3, 2 + 2, 1 - 2)$

$=< 1, 4, -1 >$

$$\begin{bmatrix} i & j & k \\ 0 & 4 & 2 \\ 1 & 4 & 1 \end{bmatrix} = i\begin{bmatrix} 4 & 2 \\ 4 & 1 \end{bmatrix} + j\begin{bmatrix} 0 & 2 \\ 1 & 1 \end{bmatrix} + k\begin{bmatrix} 0 & 4 \\ 1 & 4 \end{bmatrix}$$

$= i(4-8)+j(0-2)+k(0-4)$
$= -4i-2j-4k \rightarrow < -4,-2,-4 >$

$$-4(x-3)-2(y+2)-4(Z-2)=0$$

$= -4x+12-2y-4-4Z+8=0$

$\boxed{S_1: -2X-Y-2Z = -13}$ Type equation here.

43 c) Find equation of a planes S_2 whose passes through point F(1, -1, 2) and contains $\vec{r}(t) = < 2,3,1 > +t < 1,3,-2 >$

$\vec{r}(t) = < 2+t, 3+3t, 1-2t >$
$\vec{r}(0) = < 2,3,1 > = A$
$\vec{r}(1) = < 3,6,-1 >$ = B

$\overrightarrow{AF} = (2-1),(3+1),(1-2) = < 1,4,-1 >$
$\overrightarrow{BF} = (3-1),(6+1),(-1-2) = < 2,7,-3 >$

$$\overrightarrow{AF}\times\overrightarrow{BF} = \begin{bmatrix} i & j & k \\ 1 & 4 & -1 \\ 2 & 7 & -3 \end{bmatrix}$$

$= i\begin{bmatrix} 4 & -1 \\ 7 & -3 \end{bmatrix} + j\begin{bmatrix} 1 & -1 \\ 2 & -3 \end{bmatrix} + k\begin{bmatrix} 1 & 4 \\ 2 & 7 \end{bmatrix}$
$= i(-12+7)+j(-3+2)+k(7-8)$
$= i(-5)+j(-1)+k(-1)$

$= 5(x-1)-(y+2)-(z-2)=0$
$\boxed{S_2: 5x-y-z=5}$

43 d) Find equation of the plane S_3 whose contains $\vec{r_1}(t) = < -1,1,1 > +t < 2,-2,3 >$ *and* $\vec{r_2}(t) = < 3,2,1 > +t < -2,1,1 >$.

$x: -1+2t = 3-2t \quad 4t = 4 \quad t=1 \quad x = 1$
$y: 1-2t \quad 1-2(1) \quad y=-1$
$z: 1+3t \quad 1+3(1) \quad z = 4$
$P = (1,-1,4)$

$$n = < 2,-2,3 > \times < -2,1,1 > = < -4,-2,3 >$$

$S_3 = -4(x-1)-2(y+1)+3(z-4)=0$
$= -4x+5-2y-2+3z-12$
$= -4x-2y+3z-9 \qquad \boxed{S_3: -4x-2y+3z=9}$

43 e) Find point of intersection of line L (from #43a) and each of the planes (From #43b, #43c and #43d)

$Line\ L: \mathrm{X} = 1 + \mathrm{t}, \mathrm{Y} = -1 - \mathrm{t}, \mathrm{Z} = 2 + \mathrm{t}$
$S_1: -2X - Y - 2Z = -13$
$S_2: 5x - y - z = 5$
$S_3: -4x - 2y + 3z = 9$

$P_1 = S_1 at\ Line\ L: -2(1+t) - (-1-t) - 2(2+t)$ = -13
$\rightarrow -2 - 2t + 1 + t - 4 - 2t = -13 \qquad t = \frac{8}{3}$

$$P_1 = \left(1 + \frac{8}{3}\right), \left(-1 - \frac{8}{3}\right), \left(2 + \frac{8}{3}\right) = \boxed{\left(\frac{11}{3}, -\frac{11}{3}, \frac{14}{3}\right)}$$

$P_2 = S_2 at\ Line\ L: -5(1+t) - (-1-t) - (2+t)$ = 5
$\rightarrow -5 - 5t + 1 + t - 2 - t$ = 5
$\rightarrow t = -\frac{11}{5}$
$P_2 = \left(1 - \frac{11}{5}\right), \left(-1 + \frac{11}{5}\right), \left(2 - \frac{11}{5}\right)$
$= \boxed{\left(-\frac{6}{5}, \frac{6}{5}, -\frac{1}{5}\right)}$

$P_3 = S_3\ at\ Line\ L: -4(1+t) - 2(-1-t) + 3(2+t)$ = 9
$\rightarrow -4 - 4t + 2 + 2t + 12 + 6t$ = 5
$\rightarrow t = -\frac{5}{4}$
$P_3 = \left(1 - \frac{5}{4}\right), \left(-1 + \frac{5}{4}\right), \left(2 - \frac{5}{4}\right)$
$= \boxed{\left(-\frac{1}{4}, \frac{1}{4}, \frac{3}{4}\right)}$

43 f) Find line of intersection of $S_1\ and S_2. Call\ the\ line\ L_{3.}$

$S_1: 2X + Y + 2Z = 13$
$S_2: 5x - y - z = 5$

Assume Z = 0

$S_1: 2x + y = 13$
$\underline{S_2: 5x - y = 5}$
$7x = 18$
$x = \frac{18}{7}$

$y: 2\left(\frac{18}{7}\right) + y = 13$

$y = \frac{55}{7}$

$n(S_1)\times n(S_2) = \begin{bmatrix} i & j & k \\ 2 & 1 & 2 \\ 5 & -1 & -1 \end{bmatrix}$

= $i\begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix} + j\begin{bmatrix} 2 & 2 \\ 5 & -1 \end{bmatrix} + k\begin{bmatrix} 2 & 1 \\ 5 & -1 \end{bmatrix}$

= $i(1) - j(12) - k(7)$

= $< 1, -12, -7 >$

$$\boxed{L_3 = \left(t + \tfrac{18}{7}\right), \left(-12t + \tfrac{55}{7}\right), (-7t)}$$

43 g) Find line of intersection of S_2 $and S_3. Call\ the\ line\ L_1.$

$S_2: 5x - y - z = 5$
$S_3: -4x - 2y + 3z = 9$

Assume Z = 0

$S_2: -10x + 2y = -10$
$S_3: -4x - 2y = 9$

$-14x = -1$

$x = \frac{1}{14}$

$y: 5\left(\frac{1}{14}\right) - y = 9$

$y = -\frac{121}{14}$

$n(S_2)\times n(S_3) = \begin{bmatrix} i & j & k \\ 5 & -1 & -1 \\ -4 & -2 & 3 \end{bmatrix}$

= $i\begin{bmatrix} -1 & -1 \\ -2 & 3 \end{bmatrix} + j\begin{bmatrix} 5 & -1 \\ -4 & 3 \end{bmatrix} + k\begin{bmatrix} 5 & -1 \\ -4 & -2 \end{bmatrix}$

= $-i(5) + j(11) - k(6)$

= $< -5, 11, -6 >$

$$\boxed{L_1 = \left(-5t + \frac{1}{14}\right), \left(11t - \frac{121}{14}\right), (-6t)}$$

$$n(S_2)\times n(S_3) = \begin{bmatrix} i & j & k \\ 5 & -1 & -1 \\ -4 & -2 & 3 \end{bmatrix}$$

= $i\begin{bmatrix} -1 & -1 \\ -2 & 3 \end{bmatrix} + j\begin{bmatrix} 5 & -1 \\ -4 & 3 \end{bmatrix} + k\begin{bmatrix} 5 & -1 \\ -4 & -2 \end{bmatrix}$

= $-i(5) + j(11) - k(6)$

= $< -5,11,-6 >$

$$\boxed{L_1 = \left(-5t + \frac{1}{14}\right), \left(11t - \frac{121}{14}\right), (-6t)}$$

43 h) Find the line of intersection of $S_1 and\ S_3 call\ the\ line\ L_2.$

$S_1: 2x + y + 2z = 13$
$S_3: -4x - 2y + 3z = 9$

Assume Y = 0

$S_1: 4x + 4z = 36$
$S_3: -4x + 3z = 9$

$7z = 45$
$z = \frac{45}{7}$

$x: 4x - \frac{45}{7} = 9$
$x = \frac{27}{7}$

$$n(S_1)\times n(S_3) = \begin{bmatrix} i & j & k \\ 2 & 1 & 2 \\ -4 & -2 & 3 \end{bmatrix}$$

= $i\begin{bmatrix} 1 & 2 \\ -2 & 3 \end{bmatrix} + j\begin{bmatrix} 2 & 2 \\ -4 & 3 \end{bmatrix} + k\begin{bmatrix} 2 & 1 \\ -4 & -2 \end{bmatrix}$

= $-i(7) + j(14) - k(0)$

= $< 7,14,0 >$

$$\boxed{L_2 = \left(7t + \frac{27}{7}\right), (14t), (0)}$$

43 i) *Show that any of the lines crosses the other planes exactly at the same point*

$S_1: 2X + Y + 2Z = 13$
$S_2: 5x - y - z = 5$

$S_3: -4x - 2y + 3z = 9$

$L_3: \left(t + \frac{18}{7}\right), \left(-12t + \frac{55}{7}\right), (-7t)$

$L_3\ through\ S_1$:

$2\left(t + \frac{18}{7}\right) - 12t + \frac{55}{7} - 14t = 13$

$2t + \frac{36}{7} - 12t + \frac{55}{7} - 14t = 13$

$-24t = 13 - \frac{36}{7} - \frac{55}{7}$

$t = 0$

$\boxed{Point\ L_3S_1\left(\frac{18}{7}, \frac{55}{7}, 0\right)}$

$L_3\ through\ S_2$:

$5\left(t + \frac{18}{7}\right) - \left(-12t + \frac{55}{7}\right) - (-7t) = 5$

$5t + \frac{53}{7} + 12t - \frac{55}{7} + 7t = 5$

$24t = 5 + \frac{2}{7}$

$t = \frac{37}{168}$

$\boxed{Point\ L_3S_2\left(2\frac{19}{24}, 5\frac{3}{14}, 1\frac{13}{24}\right)}$

$L_3\ through\ S_3$:

43 j)

$$\theta_1 = \sin^{-1}\frac{\vec{L}_1 \times \vec{L}_2}{|\vec{L}_1||\vec{L}_2|} = \frac{\begin{vmatrix} -5 & 11 & -6 \\ 7 & 14 & 0 \end{vmatrix}}{\sqrt{5^2 + 11^2 + 6^2}\left(\sqrt{7^2 + 14^2 + 0^2}\right)}$$

$$= \sin^{-1}\frac{21}{\sqrt{182}\sqrt{210}}$$

$$\theta_2 = \sin^{-1}\frac{\vec{L}_1 \times \vec{L}_3}{|\vec{L}_1||\vec{L}_3|} = \frac{\begin{vmatrix} -5 & 11 & -6 \\ 1 & -12 & -7 \end{vmatrix}}{\sqrt{5^2 + 11^2 + 6^2}\left(\sqrt{1^2 + 12^2 + 7^2}\right)}$$

$$= \sin^{-1}\frac{179}{\sqrt{182}\sqrt{194}}$$

$$\theta_3 = \sin^{-1}\frac{\vec{L}_2 \times \vec{L}_3}{|\vec{L_2}||\vec{L}_3|} = \frac{\begin{vmatrix} 7 & 14 & 0 \\ 1 & -12 & -7 \end{vmatrix}}{\sqrt{7^2+14^2+0^2}\left(\sqrt{1^2+12^2+7^2}\right)}$$

$$= \sin^{-1}\frac{245}{\sqrt{210}\sqrt{194}}$$

43 k)

$S1 < -4, -2, -4 >$
$S2 \ < -5, -1, -1 >$
$S3 \ < -4, -2, 3 >$

$$\theta_1 = \sin^{-1}\frac{\vec{S}_1 \times \vec{S}_2}{|\vec{S_1}||S_2|} = \frac{\begin{vmatrix} -4 & -2 & -4 \\ -5 & -1 & -1 \end{vmatrix}}{\sqrt{4^2+-2^2+-4^2}\left(\sqrt{-5^2+-1^2+-1^2}\right)}$$

$$= \sin^{-1}\frac{24}{\sqrt{36}\sqrt{27}}$$

$$\theta_2 = \sin^{-1}\frac{\vec{S}_1 \times \vec{S}_3}{|\vec{S_1}||S_3|} = \frac{\begin{vmatrix} -4 & -2 & -4 \\ -4 & -2 & 3 \end{vmatrix}}{\sqrt{4^2+-2^2+-4^2}\left(\sqrt{4^2+2^2+3^2}\right)}$$

$$= \sin^{-1}\frac{32}{\sqrt{36}\sqrt{29}}$$

$$\theta_3 = \sin^{-1}\frac{\vec{S}_2 \times \vec{S}_3}{|\vec{S_2}||S_3|} = \frac{\begin{vmatrix} -5 & -1 & -1 \\ -4 & -2 & 3 \end{vmatrix}}{\sqrt{-4^2+-2^2+3^2}\left(\sqrt{-5^2+-1^2+-1^2}\right)}$$

$$= \sin^{-1}\frac{22}{\sqrt{27}\sqrt{29}}$$

43 l) IN PROGRESS

43 m) IN PROGRESS

43 n)

Points: A(2,1,-1) B(3,2,1) C(3,1,0)

$\overrightarrow{AB} = (3-2), (2-1), (-1+1) =< 1,1,0 >$
$\overrightarrow{AC} = (3-2), (1-1), (0+1) =< 1,0,1 >$

$$A = \frac{1}{2}\begin{vmatrix} i & j & k \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{vmatrix} = \frac{1}{2}(1+1-1)$$

$\boxed{A = \frac{1}{2}}$

43 o)

Points: A(1,2,-1) B(2-1,3), C(2,-1,2) and D(2,1,2)

$\overrightarrow{AB} = (2-1), (1-2), (3+1) = < 1, -1, 4 >$
$\overrightarrow{AC} = (2-1), (-1-2), (2+1) = < 1, -3, 3 >$
$\overrightarrow{AD} = (2-1), (1-2), (2+1) = < 1, -1, -3 >$

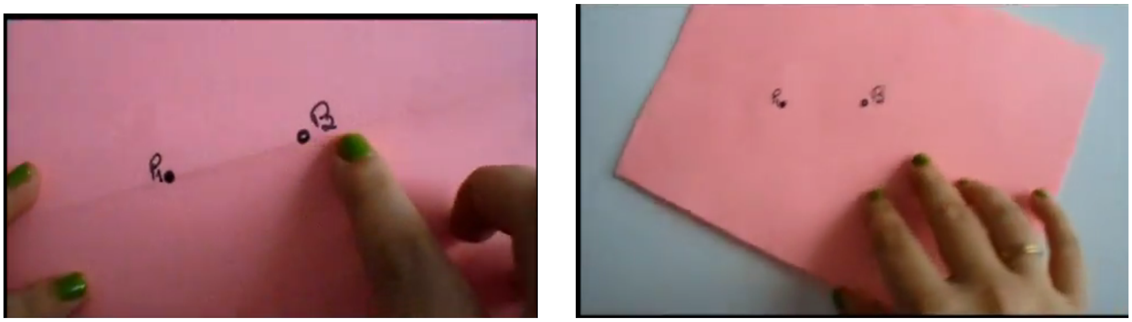
$A = \begin{vmatrix} 1 & -1 & 4 \\ 1 & -3 & 3 \\ 1 & -1 & -3 \end{vmatrix} = 6\begin{vmatrix} -3 & 3 \\ -1 & -3 \end{vmatrix} + 1\begin{vmatrix} 1 & 3 \\ 1 & -3 \end{vmatrix} + 4\begin{vmatrix} 1 & -3 \\ 1 & -1 \end{vmatrix}$
$= 6(9+3) + (-3-3) + 4(-1+3) = \ 74$

$\boxed{Volume = 74}$

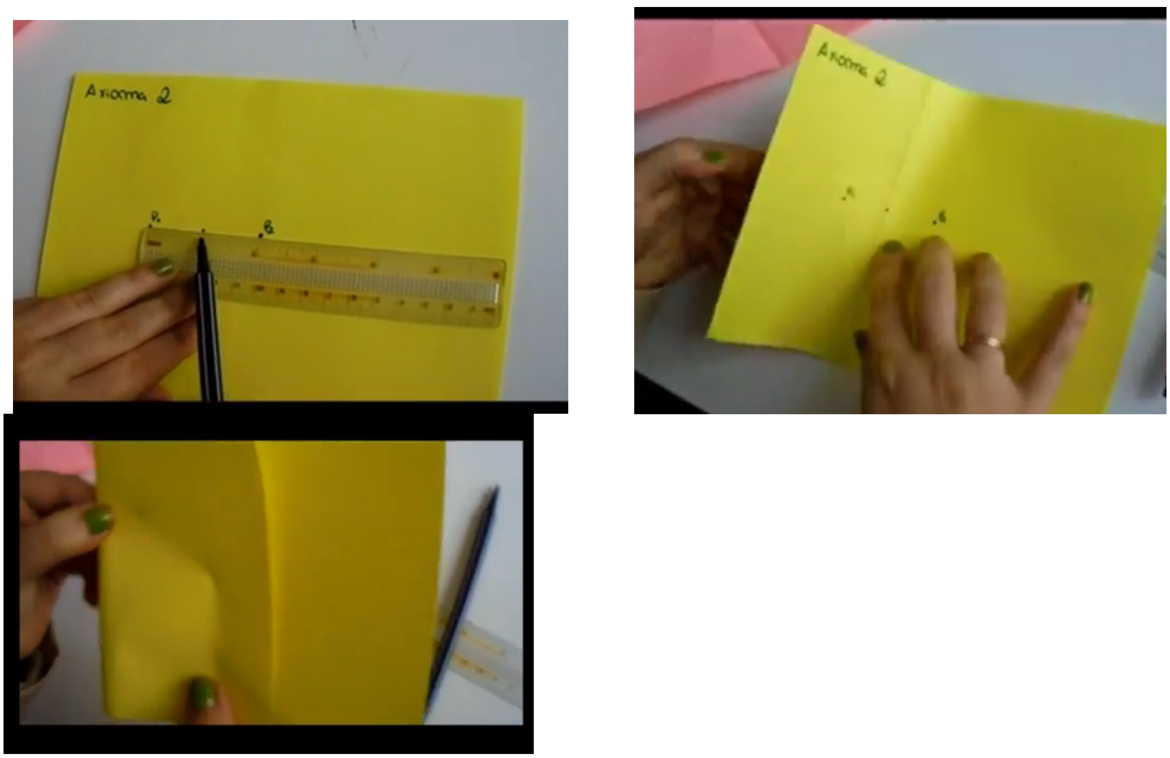
44- Go through all the 7 Geometric theorems and proof those using vectors. VECTOR quantities are defined by a magnitude and a direction. Examples are velocity, acceleration and displacement. Both the magnitude and the direction would be stated.

45- Use a piece of paper to demonstrate all the axioms in origami.

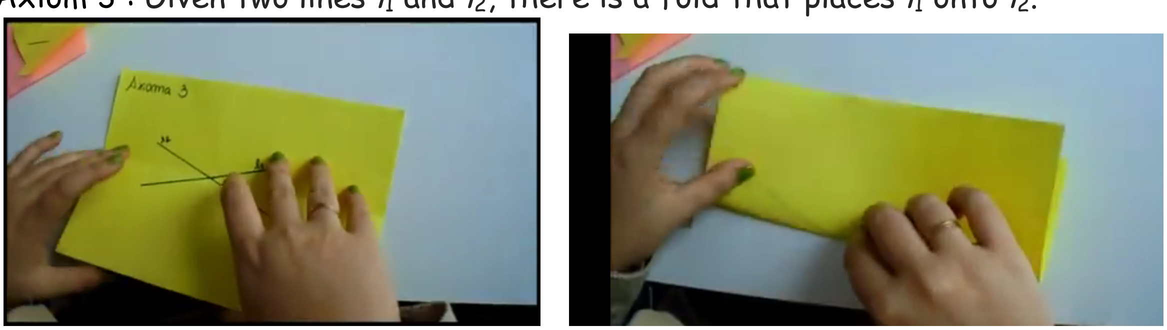
Axiom 1: Given two points p_1 and p_2, there is a unique fold that passes through both of them.

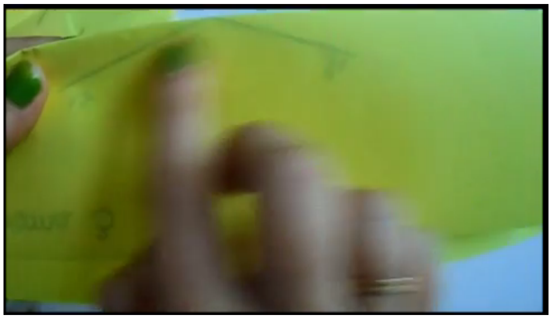


Axiom 2: Given two points p_1 and p_2, there is a unique fold that places p_1 onto p_2.

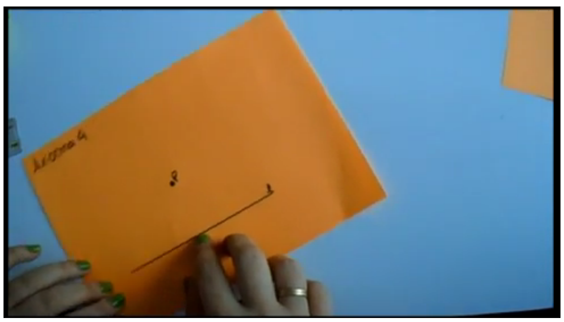


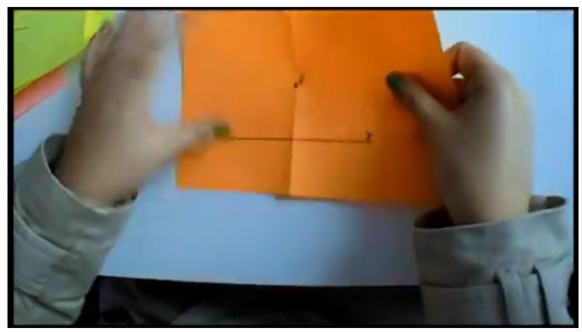
Axiom 3 : Given two lines l_1 and l_2, there is a fold that places l_1 onto l_2.



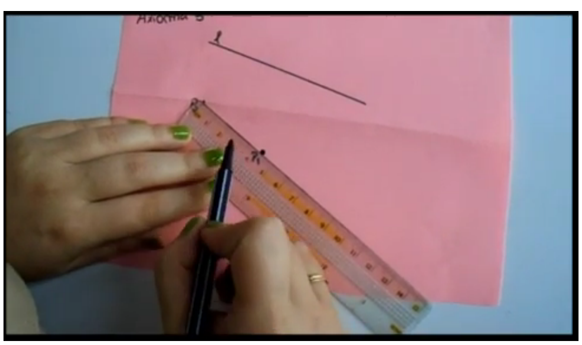


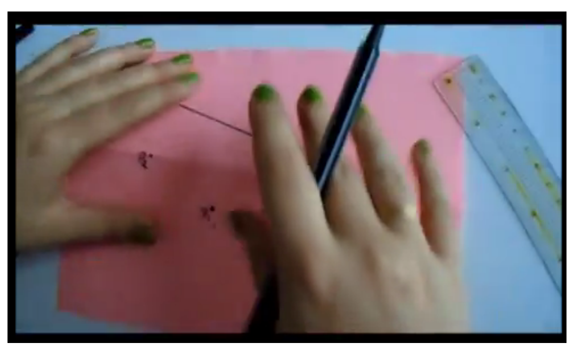
Axiom 4: Given a point p_1 and a line l_1, there is a unique fold perpendicular to l_1 that passes through point p_1.

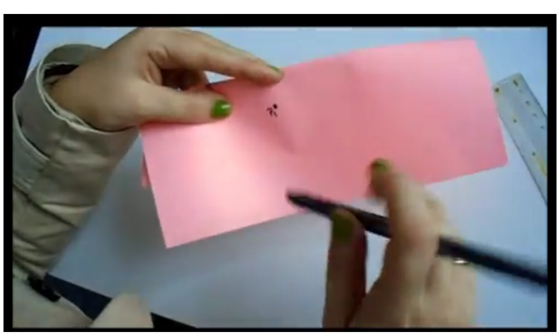




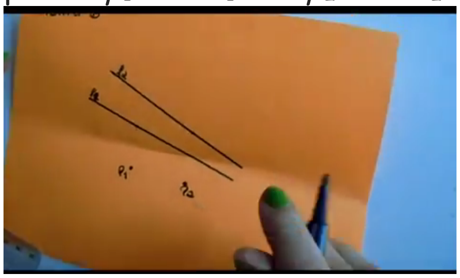
Axiom 5: Given two points p_1 and p_2 and a line l_1, there is a fold that places p_1 onto l_1 and passes through p_2.

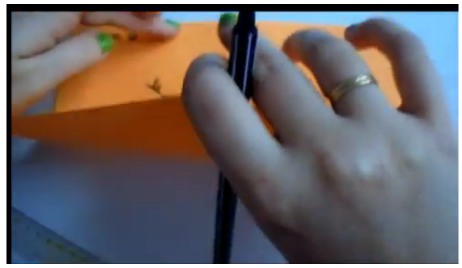




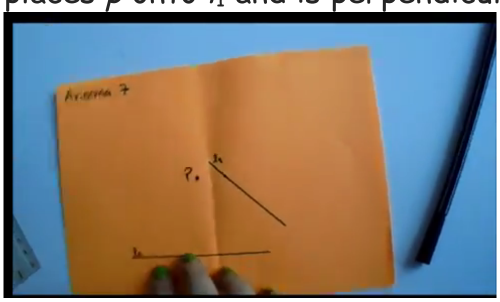


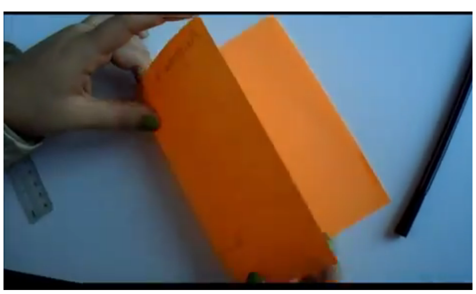
Axiom 6: Given two points p_1 and p_2 and two lines l_1 and l_2, there is a fold that places p_1 onto l_1 and p_2 onto l_2.

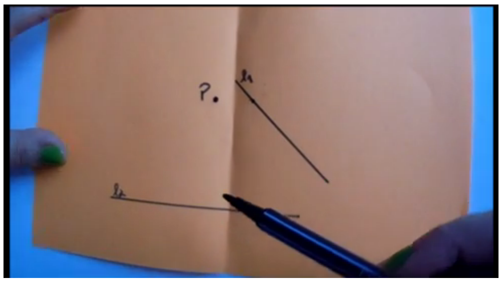




Axiom 7: Given one point p and two lines l_1 and l_2, there is a fold that places p onto l_1 and is perpendicular to l_2.







46- Show that $\frac{d}{d\theta}U_r = U_\theta$ *and* $\frac{d}{d\theta}U_\theta = -U_r$ then show that

$\{\vec{R} = rU_r \qquad \vec{V} = r\dot{\theta}U_\theta + \dot{r}U_r \qquad \vec{a} = (\ddot{r} - r\dot{\theta}^2)U_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})U_\theta$

$a\,)\ \frac{d}{d\theta}\,U_r = U_\vartheta$

$U_r = (\cos\theta\,, \sin\theta)$

$U_\vartheta = (-\sin\theta\,, \cos\theta)$

$\frac{d}{d\theta}\,U_r = (-\sin\theta\,, \cos\theta)$

$b)\ U_\vartheta = (-\sin\theta\,, \cos\theta)$

$\frac{d}{d\theta}\,U_\theta = (-\cos\theta\,, -\sin\theta)$

$\frac{d}{d\theta}\,U_\theta = -U_r$

c)

$C)\ \vec{R} = \int U_r dr = rU_r dr + C\ where\ C\ is\ 0$

$= rU_r dr$

d)

$$\vec{V}=\frac{\mathrm{d}}{dt}\vec{R}=r\cdot\frac{d}{dt}\cdot U_r+\dot{r}\cdot U_r=r\dot{\theta}U_\theta+\overset{*}{r}U_r$$

$$where...\frac{d}{dt}U_r=U_\theta\,\dot{\theta}$$

e)

$$\vec{a}=\dot{r}\dot{\theta}U_\theta+r\ddot{\theta}U_\theta+r\dot{\theta}\frac{d}{dt}U_\theta+\ddot{r}U_r+\dot{r}\frac{d}{dt}U_r$$

$$where...\frac{d}{dt}U_\theta=-U_r\dot{\theta}$$

47) Show that $(\mathrm{r}^2\dot{\theta})=constant, then, a_r=\frac{-K}{r^2}$

a)

$$a_\theta=(r\ddot{\theta}+2\dot{r}\dot{\theta})=\frac{r^2\ddot{\theta}+2r\dot{r}\dot{\theta}}{r}=\frac{1}{r}\frac{d}{dt}(r^2\dot{\theta})$$

Solution:

Since $a_\theta=0$ then $\frac{d}{dt}(r^2\dot{\theta})=0$ $(r^2\dot{\theta})=Const$ then $\dot{\theta}=\frac{c}{r^2}$

b)

$$a_r=(\ddot{r}-r\dot{\theta}^2)=\frac{-K}{r^2}$$ since $\dot{r}=\frac{dr}{dt}=\frac{dr}{d\theta}\frac{d\theta}{dt}=\frac{dr}{d\theta}\frac{c}{r^2}$ Let $z=\frac{1}{r}$ $\frac{dz}{d\theta}=\frac{-1dr}{r^2d\theta}$

$$\cdot\ \dot{r}=-c\frac{dz}{d\theta}$$

With the above substitution we get This sub eliminate from r^2 the equation

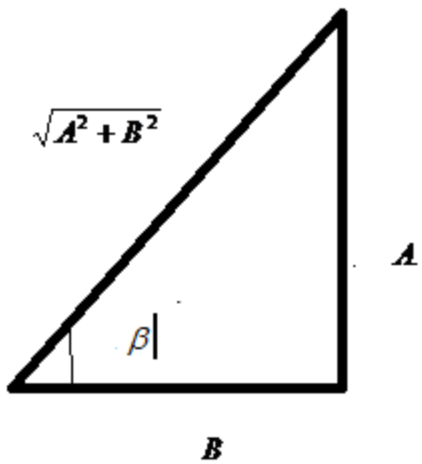
$$\ddot{r}=-c\frac{d^2z}{d\theta^2}\frac{c}{r^2}=-\frac{c^2}{r^2}\frac{d^2z}{d\theta^2}=-c^2z^2\frac{d^2z}{d\theta^2}$$ Rewrite $a_r=\ddot{r}-r\dot{\theta}^2=-c^2z^2\frac{d^2z}{d\theta^2}-c^2z^3$

$$a_r=-c^2z^2(\frac{d^2z}{d\theta^2}+z)=-Kz^2$$

$z = \frac{1}{r}$

So finally we get Solution: $a_r = (\ddot{r} - r\dot{\theta}^2) = \frac{-K}{r^2}$

48- Show that $A\sin\theta + B\cos\theta = H\cos(\theta - \beta)$ the value of $H = \sqrt{A^2 + B^2}$



$A = \sqrt{A^2 + B^2}\sin\beta \qquad B = \sqrt{A^2 + B^2}\cos\beta$ plug into the original function

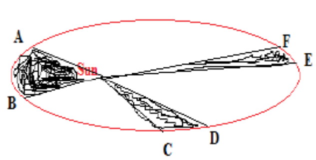
$A\sin\theta + B\cos\theta = H\cos(\theta - \beta)$ then we get

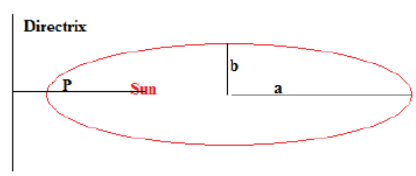
$\sqrt{A^2 + B^2}\sin\beta\sin\theta + \sqrt{A^2 + B^2}\cos\beta\cos\theta = \sqrt{A^2 + B^2}(\cos\theta\cos\beta + \sin\beta\sin\theta) = \sqrt{A^2 + B^2}(\cos\theta - \beta)$

$= H\cos(\theta - \beta)$

Solution: Finally We get : $H = \sqrt{A^2 + B^2}$

49- Show the following relation in an ellipse $b^2 = a^2\ (1 - e^2)$ and $Pe = a(1 - e^2)$





1)

As you know $b^2 = a^2 - b^2, where, e = \frac{c}{a}$

Thus , we get $b^2 = a^2(1 - \left(\frac{c}{a}\right)^2)$ Then we have to simplify this equation:

Thus, we get Solution $b^2 = a^2(1 - e^2)$

2)

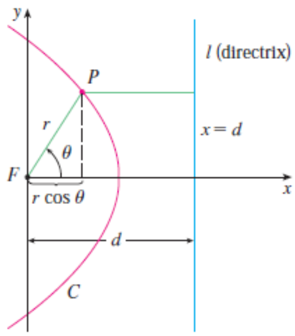


FIGURE 1

Where d =p

$|PE| = \mathrm{r}, and |PI| = d - r\cos\theta \qquad r = e(d - r\cos\theta)$

If we square both sides of this polar equation and convert to rectangular coordinates,

we get $x \ + y^2 = e^2(d^2 - 2dx + x^2), \Rightarrow (1-e^2)x + 2de^2x + y^2 = e^2d^2$

After completing the square, we have

$$\left(x + \frac{e^2}{1-e^2}\right)^2 + \left(\frac{y^2}{1-e^2}\right) = \left(\frac{e^2d^2}{(1-e^2)^2}\right)$$

-If , we recognize Equation 3 as the equation of an ellipse. In fact, it is of the form

$$\left(x + \frac{e^2}{1-e^2}\right)^2 \times \left(\frac{(1-e^2)^2}{e^2d^2}\right) + \left(\frac{y^2}{1-e^2}\right) \times \left(\frac{(1-e^2)^2}{e^2d^2}\right) = \frac{e^2d^2}{(1-e^2)^2} \times \left(\frac{(1-e^2)^2}{e^2d^2}\right)$$

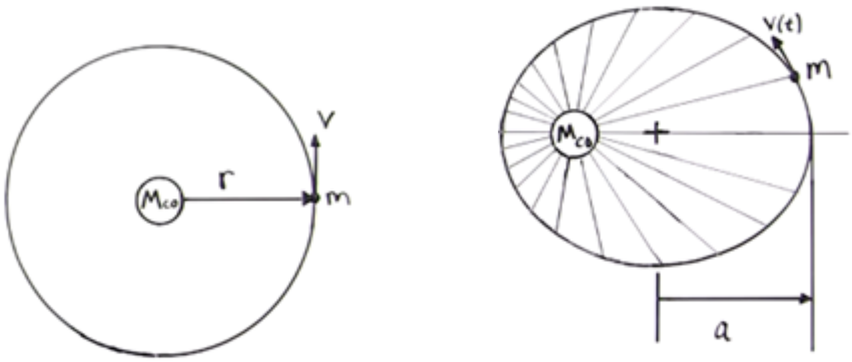
Part 1 Becomes $\left(x + \frac{e^2d}{(1-e^2)}\right)^2 + \frac{y^2}{(1-e^2)} = \frac{e^2d^2}{(1-e^2)^2}$

$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ let look at the part one $\frac{(x-h)^2}{(a^2)}$

Simplify : $\left(\frac{x + \frac{e^2d^2}{1-e^2}}{\frac{e^2d^2}{(1-e^2)^2}}\right)^2 = \frac{(x-h)^2}{(a^2)}$, we get $a^2 = \frac{e^2d^2}{(1-e^2)^2}$

Since d=p, we can simplify to get solution: $Pe = a(1-e^2)$

50- Show the following relation $\frac{T^2}{a^3} = \frac{4\pi^2}{GM}$ between the period and mean distance of the
planet to the sun.



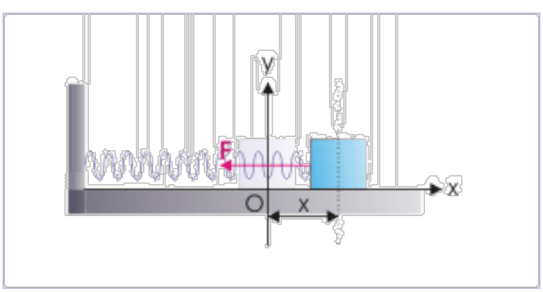
$F_c = ma. \Rightarrow\Leftarrow$ equal to $\frac{GMm}{r^2}$

$\frac{GMm}{r^2} = m\Omega^2 r$ $\quad \frac{1}{rm} \times \frac{GMm}{r^2} = m\Omega^2 r \times \frac{1}{rm}$ Thus we get $\frac{GM}{r^3} = \Omega^2$ where $\Omega = (\frac{2\pi}{T}), \rightarrow\rightarrow\rightarrow$
$\frac{GM}{r^3} = \frac{4\pi^2}{T^2}$ $\quad T^2 GM = 4\pi^2 r^3$

solution: $\frac{T^2}{r^3} = \frac{4\pi^2}{GM}$

51- Match that total energy for the following models of conic sections on v-x coordinates.

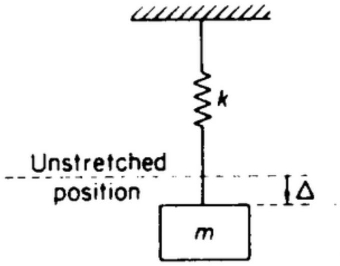
a) An oscillating spring on a horizontal plane with no friction.



$\frac{mv^2}{2E} + \frac{Kx^2}{2E} = 1$ $\quad , \frac{mv^2}{2E} + \frac{kx^2}{2E} = 1$, simplify :divde E for both sides

SOLUTION $\frac{v^2}{\left(\frac{2E}{m}\right)} + \frac{x^2}{\left(\frac{2E}{m}\right)} = 1$

b) An oscillating spring on a vertical plane (gravity) with no friction.



Kinetic enegry: $E_{total} = KE + PE$ $\frac{1}{2}mv^2 + mgx = E$ simplify :divde E for both sides

$$KE = \frac{1}{2}\mathrm{MV}^2, PE = Mgx, \rightarrow\rightarrow\rightarrow E_{\mathrm{total}} = \frac{v^2}{\left(\sqrt{\frac{2E}{m}}\right)^2} + \frac{x}{\left(\frac{E}{mg}\right)} = 1$$

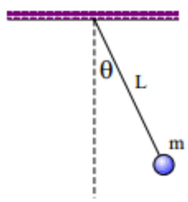
Potential energy: PE=mgx; where x is the height
E=total energy: E=PE+KE
where y is the height

$$\frac{v^2}{\left(\sqrt{\frac{2E}{m}}\right)^2} + \frac{x}{\left(\frac{E}{mg}\right)} = 1$$

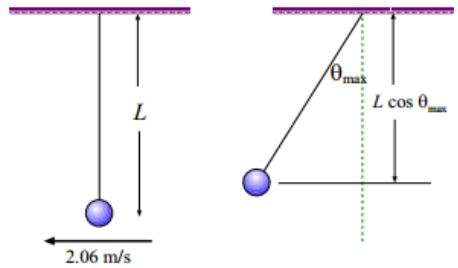
Solution:

c) An oscillating pendulum on a vertical plane with no friction

RKED EXAMPLES



(a)



(a) (b)

$$E_{total} = KE + PE \quad ; \quad KE = \frac{1}{2}mv^2, PE = mgy \qquad E = \frac{1}{2}\mathrm{mv}^2 + mgy$$

simplify :divde E for both sides

$$\frac{mv^2}{2E} + \frac{2mgy}{2E} = 1$$

Solution: $$\frac{v^2}{\left(\sqrt{\frac{2E}{m}}\right)^2} + \frac{y^2}{\left(\sqrt{\frac{E}{mg}}\right)^2} = 1$$