Homework set Chapter 11

1- For line $l:\ p(3,2,-2)\ and\ \vec{V} =< 1,-1,2 >$ and the plane S which contains points A(1,2,1), B(2,-1,2), and C(0,-2,1).

a) Find the point of intersection of the line and the plane. (time and space)

b) Find the acute angle between the line and the plane

c) How far the position of a particle on the line at $t = 2\sec$ is away from the plane?

d) If the source of light is shines normal to the plane, how fast the shadow of the particle is moving on the plane?

2- For the given Planes $S_1 : 2x + y - z = 4$ and $S_2 : x + 2y + z = -1$

a) Find line of intersection of the planes

b) Find the angle between the planes

c) Graph the planes and indicate the line of intersection and the angle

d) If a particle on plane S_1 at point (1,1,-1) moves directly to pint H on line of intersection of the planes, what are the coordinates of point H?

e) If the point P(1,1,-1)shines a laser beam in the direction of $\vec{V} =< 1,-1,1 >$, what are the coordinate of point (Q) intersection of laser beam with plane S_2?

f) Find the Area of a triangle made by point P, Q, and H as its vertices?

3- Solve the systems of equations by finding point of intersection of the planes.

$$\begin{cases} x+2y-z=3 \\ 2x+y+z=4 \\ 2x-y+2z=5 \end{cases}$$

First find line of intersection of two planes and then use the line and the third plane to find the point of intersection.

4-A Laser beam at point P (1,2,1) shines on to mirror S: $2x+y-2z=6$,in the direction of $\vec{V}=<2,2,1>$. We want to locate position of a detector after 2sec which the beam bounces off of the plane. How far this point is away from point P(1,2,1)?

5- A Laser beam at point P (1, 1, -1) is aimed at point Q (-2, -1, 0) after it bounces off of mirror S: $x+y-z=4$. What is the location on the mirror which the beam bounces off of?

6- Given plane S: $x+y+z=2$, Graph it

a) Find the volume created by plane S and the coordinate planes in the first octane?

b) Find the distance of the plane $x+y+z=2$ to the origin.

7- Find a general formula for

a) The distance of a point to a plane. (2 different methods)

b) The distance between two parallel planes. (2 different methods)

c) The distance between two skewed lines.

d) The distance between a point and a line. (4 different methods)

8- Find the equation of tangent plane to the sphere with equation C: $x^2 + y^2 + z^2 = 9$at point P(2, 2, 1).

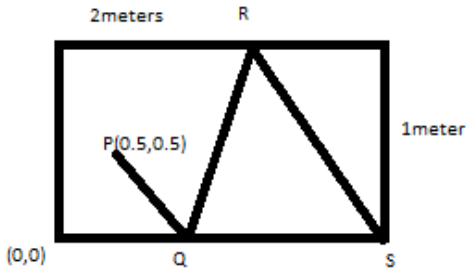
9- Find the parametric equation of line of intersection between two tangent planes to the sphere with equation C: $x^2 + y^2 + z^2 = 9$at point P(1, -2, 2) and Q(1,2,2).

10- Show that a) the three points A (1,2,-1) , B (2,-1,1) and C(3,-4,3) are collinear and find the line which passes through them. B) the four points A (1,2,1) , B (0,-1,1) , C(1,-4,2) and D(2,1,0) are not coplanar. Then find the distance of each point to the plane created by the other three points.

11- Find the relation between magnitude Dot product and Cross product.

12- Show that sum of the squares of the directional cosines is one.

13- You want to place the ball in to pocket at point S, while it bounces at point Q and R. What are the locations of point Q and R on the rim of the pool table as shown?



14- Find the equation of the plane that passes through the point (-1, 2, 1) and contains the line of intersection of the planes $x + y - z = 2$ and $2x - y + 3z = 1$.

15-Given a system of equations.

$$2x + y - z = 2$$
$$x + 2y + z = 1$$
$$2x - y - 2z = 3$$

a) Solve for line of intersection of first two planes.

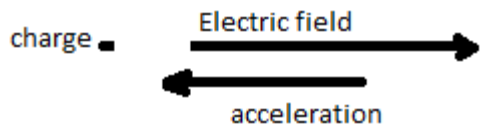
b) Solve for the point of intersection of above line with the third plane.

16- Graph $x + y + z = 4$ and Find the closest point on the above plane to point (-1, -3, 2)

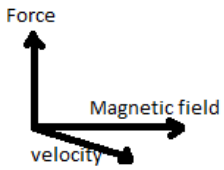
17- Graph $2x + y - z = 2$ Find the shortest distance from the plane to the origin in 4 different methods.

1) Partial derivatives. 2) Dot product. 3) Cross product. 4) Geometry

18-When a charge particle moves in an electric field. The force excreted on the charge particle is $\sum \vec{F} = q\vec{E}$ where "q" is the charge and $\vec{E}$ is the electric field (vector). We know that $\sum \vec{F} = m\vec{a}$ then the $m\vec{a} = q\vec{E}$ can be used to find the mass of a charge particle if the electric field, acceleration and charge of the particle are known. Find the mass of a charged particle with $q = -2$ Columb in a field $\left|\vec{E}\right| = \sqrt{52} \times 10^{10}\, N/C$ and acceleration $\vec{a} = <-16,\ -24> 10^{20}\, m/s^2$.



19- When a charge particle moves in a magnetic field. The force excreted on the charge particle is $\sum \vec{F} = q\ (\vec{v} \times \vec{B})$ where "$\vec{v}$"is the velocity of the particle and $\vec{B}$ is the Magnetic field (vector). We know that $\sum \vec{F} = m\vec{a}$ then the $m\vec{a} = q(\vec{v} \times \vec{B})$ can be used to find the mass of a charge particle if the magnetic field, acceleration, velocity and charge of the particle are known. Find acceleration of a charged particle with $m = 5 \times 10^{-10}\, Kg\ \ and\ \ q = -5 \times 10^{-6}\ \ C$ in a field $\vec{B} =< 10,\ \ 20 > Tesla$ and velocity $\vec{v} =< 8,\ \ -4 > 10^{6}\, m/s$. Find angle between acceleration / velocity and acceleration / magnetic field.



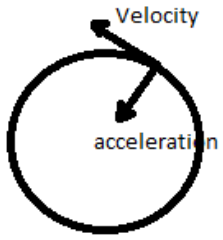
20-When a particle moves in straight line, and then its kinetic energy is $KE = \frac{1}{2} m\vec{v} \bullet \vec{v}$

What is KE (in Joules) of a 2miligram fly that is going with velocity $\vec{v} =< 4,\ \ 3 > m/s$?

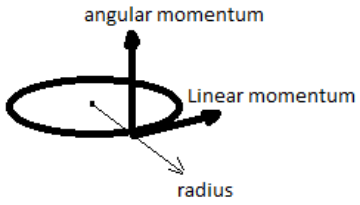
21-Potential energy of an object at height H is $PE = m\vec{g}. \bullet \Delta\vec{H}$ where" g "is acceleration of gravity and ΔH is change in height. A 2kg mass moved from point A(3, -4) to point B(5, 12). How much work was done on the object? $\vec{g} =< 0, -10 > m/s^2$

22- Central petal force on an object is a fictitious force do to rotation. If a mass moves around a circle of radius R with linear speed of $\vec{v}$, then the magnitude of the central petal

force will be $\left|\vec{F}\right| = m\frac{\vec{v} \bullet \vec{v}}{R}$ and the direction is toward the center of circle. What is acceleration (direction and magnitude) of a mass going around a circle $(R = 0.5m)$and $\vec{v} =< -5,\ 12 > m/s$. Show that acceleration is inversely proportional to the radius.



23- Angular momentum of an object is $\vec{L} = \vec{r} \times \vec{P}$where change in linear momentum for constant mass is $\Delta\vec{P} = m\Delta\vec{v}$. Find $\vec{L}$ for a 0.5 kg object going around a circle $\vec{r} =< 3,\ 2 > m \ \ and \ \ \vec{v} =< -10,\ 6 > m/s$



24- Angular momentum in 3D: Find $\vec{L}$ for a 0.5 kg object going around a circle $\vec{r} =< 3,\ 4\ \ -2 > m \ \ and \ \ \vec{v} =< 8,\ 4,\ 6 > m/s$

25- Find acceleration of a charged particle with $m = 5\times10^{-10}\,Kg \ \ and \ \ q = -5\times10^{-6}\ C$ in a field $\vec{B} =< 10,\ 20,\ 5 > Tesla$ and velocity $\vec{v} =< 8,\ -4,\ 12 > 10^6\,m/s$. Find angle between acceleration / velocity and acceleration / magnetic field.

26- Find area of a triangle with vertices A(2, 3,6) and B(3, 2, 5) and C(1, 3,4).

27- Find volume of a tetrahedron created by $\vec{V} =< 3,\ 2,\ 5 >, \vec{U} =< 3,\ 2,\ 6 >, \vec{W} =< 1,\ 4,\ 6 >$

28- Find the net force for $\vec{F}_1 =< -3,\ 5,\ -6 > N, \vec{F}_{32} =< 4,\ 2,\ 7 > N\ \vec{F}_3 =< 8,\ 4\ 4 > N$

$, \vec{F}_4 =< -5,\ 1,\ -2 > N$. Then find the acceleration of a 2kg object which influences by these forces.

29- Prove the formula for dot product and cross product.

30- a) Find the line of intersection of $2x + y - z = 2 \quad and \quad x + 2y + z = 1$

b) Find the acute angle of the above planes.

c) Find point of intersection of the above planes with plane $x - y - z = 3$.

31- Find the closest point on plane $x + y + z = 4$to the origin.

32-Find the line of intersection of $2x + y - z = 2 \quad and \quad x + 2y + z = 1$ then Find the acute angle of the planes.

33-A particle at $t =\ 0$ started at point (4, 0, 3) in the direction $v =< -1,\ 2,\ 0 >$bounces off the surface $x - y + z = 4$.

A) Find the location of the particle after 4 sec.

B) Find the displacement during the interval [0, 4] sec.

34-A particle at $t = 0$ was detected at point (2, 1, 3). Then after it bounces off the surface $x + y + z = 3$ was observed at point (2, 2, 2). For how long the particle was missing? (How long did it take for particle to travel from one point to another?)

35- Given $r(t) =< t,\ \ln|\sec t|,\ 2 >,\ 0 \le t \le \ln 2$

a) Show that $r(t)$ is a smooth vector function.

b) Find the length of the curve from t = 0 to t = ln2.

36-A particle with initial velocity $v_0 = 2\sqrt{2}\ \ m/s$ and $\theta = \frac{\pi}{4}$ is lunched from the origin in a field with $a =< 0,\ -2,\ 0 > m/s^2$. Find all the points that the particle intersects the plane $S : x + y + z = 3$.

37-Two planes $S_1 : x + 2y + z = 4 \ \ and \ \ S_2 : x - y + z = 1$ intersect at line L.

Find the distance of the point P (1, 0, 2) to both planes and line L.

38- What is the mirror image of vector $\dot{V} < 1,2,1 >$ respect to $\dot{U} < 2,-1,2 >$

39- A boy on a skateboard is moving toward a basketball hoop at 2 m/s. The hoop is 3 meters above the ground. At time t = 0, the boy throws the ball from 2 meters above the ground at an angle of 45° when he is 6 meters away. How fast (in respect to him) does he have to throw the ball in order to make a basket?

40- Find the equation of the plane that passes through the point (-1, 2, 1) and contains the line of intersection of the planes $x + y - z = 2$ and $2x - y + 3z = 1$.

41- Find the work done to move a 4kg mass with an acceleration $\vec{a} =< 2,-3,1 >$ form point $p(2,4,5)$ to point $Q(1,-3,2)$.

42- Find magnitude of torque if the moment arm is $\vec{r} =< 2,2,-1 >$ and applied force is $\vec{F} =< 3,4,-12 >$. What is the angle between moment arm and the force in above question?

43- a)Find parametric equation of a line L whose passes through point $A(1,-1,2)$, $B(2,-2,3)$.

b) Find equation of a plane S_1 whose passes through points $C(3,-2,2)$, $D(3,2,4)$, $E(4,2,1)$.

c) Find equation of a plane S_2 whose passes through point $F(1,-1,2)$ and contains $\vec{r}(t) =< 2,3,1 > +t < 1,3,-2 >$.

d) Find equation of a plane S_3 whose contains $\vec{r}(t) =< -1,1,1 > +t < 2,-2,3 >$ and $\vec{r}(t) =< 3,2,1 > +t < -2,1,1 >$.

e) Find point of intersection of line L (from #43a) and each of the planes (from #43b,#43c,and 43d)

f) Find line of intersection of S_1 and S_2. Call the line L_3.

g) Find line of intersection of S_2 and S_3. Call the line L_1.

h) Find line of intersection of S_1 and S_3. Call the line L_2.

i) Show that any of the lines crosses the other planes exactly at the same point.

j) Find the angle between Lines L_1, L_2 and L_1, L_3 and L_3, L_2. What can you conclude?

k) Find the angle between Planes S_1 , S_2 and S_1 , $S_3$and S_3 , S_2. What can you conclude?

l) Find the distance of point P(1,4,-2) to lines L_1, L_2, and L_3.

m) Find the distance of point P(1,4,-2) to Planes S_1, S_2, and S_3

n) Find the area of ΔABC where A(2,1,-1), B(3,2,1), and C(3,1,0)

o) Find Volume of a parallelepiped with A(1,2,-1), B(2,1,3), C(2,-1,2) and D(2,1,2).

44- Go through all the 7 Geometric theorems and proof those using vectors.

45- Use a piece of paper to demonstrate all the axioms in origami.

46- Show that $\frac{d}{d\theta}U_r = U_\theta \ and \ \frac{d}{d\theta}U_\theta = -U_r$ then show that

$$\vec{R} = rU_r \qquad \vec{V} = r\dot{\theta}U_\theta + \dot{r}U_r \qquad \vec{a} = (\ddot{r} - r\dot{\theta}^2)U_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})U_\theta$$

47- Show that $(r^2\dot{\theta}) = Const$ then $a_r = \frac{-K}{r^2}$

48- Show that $ASin\theta + BCos\theta = HCos(\theta - \beta)$ for value of $H = \sqrt{A^2 + B^2}$

49- Show the following relation in an ellipse $b^2 = a^2(1-e^2)$ and $Pe = a(1-e^2)$

50- Show the following relation $\frac{T^2}{a^3} = \frac{4\pi^2}{GM}$ between the period and mean distance of the planet to the sun.

51- Match that total energy for the following models of conic sections on v-x coordinates.

a) An oscillating spring on a horizontal plane with no friction.

b) An oscillating spring on a vertical plane (gravity) with no friction.

c) An oscillating pendulum on a vertical plane with no friction.