Homework set #1

1- When a plane crosses a double cones connected at their vertex, it traces the graph of conics (Ellipse, Parabola, and Hyperbola). Find a relation between the eccentricity and the angle of the plane with horizon.

2-Given the equation $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ Show the following equations:

- The equation of an ellipse $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$

$$a = \frac{1}{2A}\sqrt{\frac{D^2C + E^2A - 4ACF}{C}} \quad b = \frac{1}{2C}\sqrt{\frac{D^2C + E^2A - 4ACF}{A}}$$

- The equation of hyperbola $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

$$a = \frac{1}{2A}\sqrt{\frac{D^2C - E^2A - 4ACF}{C}} \quad b = \frac{1}{2C}\sqrt{\frac{-D^2C + E^2A + 4ACF}{A}}$$

- The equation of Parabola $(y-k) = \frac{1}{4p}(x-h)^2$

  Where $h = (\frac{-D}{2A})$, $k = (\frac{D^2 - 4AF}{4AE})$ $p = \frac{-E}{4A}$

- The angle of axes rotation to eliminate the cross term which can be calculated by $Tan(2\theta) = \frac{B}{A-C}$

3-Find rectangular equation of an object launched with the following parametric equation $\begin{cases} x(t) = (vCos\theta)t + x_0 \\ y(t) = -4.9t^2 + (vSin\theta)t + y_0 \end{cases}$ if it launched from origin, ***use the rectangular equation*** to find a) Maximum Range b) Maximum height

Is it possible to find velocity of impact? Find it.

4-Show that the parametric equation of all three different Cycloid can be written as $\begin{cases} x(t) = a\theta - bSin\theta \\ y(t) = a - bCos\theta \end{cases}$ Then find the slope of the curve in terms of angle θ.

5-Show that the parametric equation of all epicycle can be written as

$$\begin{cases} x(t) = (a+b)Cos\theta - bCos(\frac{a+b}{b}\theta) \\ y(t) = (a+b)Sin\theta - bSin(\frac{a+b}{b}\theta) \end{cases}$$

6- Show that the parametric equation of all hypocycloid can be written as

$$\begin{cases} x(t) = (a-b)Cos\theta + bCos(\frac{a-b}{b}\theta) \\ y(t) = (a-b)Sin\theta - bSin(\frac{a-b}{b}\theta) \end{cases}$$

7-Show that polar graph of Rose and cardioid can be trace by epicycles

Problem 8,9,10,and 11 are about Cycloids

8-Graph a cycloid generated by a disc with radius one, then find the tangent line to the curve at $\theta = \pi / 2$.

9- Find arc length and the area under the curve of a cycloid after one rotation of generating disc.

10- Find Surface area of rotating a cycloid after one rotation of generating disc around x axis.

11- If you double the radius of generating disc, by what factors the Arc length, the Area and the Surface area changes?

12- Find the area inside both functions. (*Graph them*)

$r_1 = Sin(2\theta)$

$r_2 = Cos\theta$

13- Find the tangent of the angle between the tangent lines of both graphs at the point of intersection.

14- Find the arc length for the parametric equation $\begin{pmatrix} x(t) = \sin t \\ y(t) = 1 - \cos^2 t \end{pmatrix}$ $0 \le t \le \frac{\pi}{2}$

15-Given the polar function $r = Sin(t)$ and line $y = y_0 \quad where \quad y_0 < 0$

What must be y_0 such that the surface area of the function rotated around $y = y_0$ is three times bigger than the surface area rotated around y = 0.

16- Graph the function $r(\theta) = Cos(2\theta)$. Show that no part of the curve overlaps with itself.

17-Find the total area inside of curve $r^2 = 2Cos(2\theta)$ and outside $r = 1$

18-Find the total area inside of curve $r = 1 + \sin\theta$ and outside $r = 3\sin\theta$

19-Find the total area inside of the loop of $r = 2 + \sin\theta$

20-Given the equation of curvature in polar to be $\kappa = \dfrac{\left|2r'^2 - rr'' + r^2\right|}{[r'^2 + r^2]^{3/2}}$

Find the radius of the curve $r = Sin\theta$ at $\theta = \alpha$.

21-Find all the VERTICAL and HORIZONTAL slopes to the curve $r^2 = \sin\theta$

	Rectangular	Polar	Parametric
Arc length	$L = \int_a^b \sqrt{1 + f'(x)^2}\,dx$	$L = \int_a^b \sqrt{r^2 + r'^2}\,d\theta$	$L = \int_a^b \sqrt{x'^2 + y'^2}\,dt$
Surface area			
	$S = 2\pi\int_a^b f(x)\sqrt{1 + f'(x)^2}\,dx$	$S = 2\pi\int_a^b r\sin\theta\sqrt{r^2 + r'^2}\,d\theta$	$S = 2\pi\int_a^b y(t)\sqrt{x'^2 + y'^2}\,dt$
	$2\pi\int_a^b f(y)\sqrt{1 + f'(y)^2}\,dy$	$S = 2\pi\int_a^b r\cos\theta\sqrt{r^2 + r'^2}\,d\theta$	$S = 2\pi\int_a^b x(t)\sqrt{x'^2 + y'^2}\,dt$

22- Find arc length for each and arrange them from highest to lowest

$a)\ f(x) = \frac{2}{3}x^{\frac{2}{3}} \qquad 1 \le x \le 8$

$b)\ r = e^{3\theta} \qquad 0 \le \theta \le 2$

$c)\ r = 2a\sin\theta \qquad 0 \le \theta \le 2\pi$

$d)\ r = \sin^2(\frac{\theta}{2}) \qquad 0 \le \theta \le \pi$

$e)\ \begin{cases} x(t) = 2t \\ y(t) = t^2 - 1 \end{cases} \qquad -1 \le t \le 2$

$f)\ \begin{cases} x(t) = t\ \sin t \\ y(t) = t\ \cos t \end{cases} \qquad 0 \le t \le \pi$

23- Find the Surface area rotated about given axis

$a)\ f(x)=\sqrt{x}-\frac{1}{3}x^{\frac{3}{2}} \qquad 1\le x\le 3 \qquad x-axis$

$b)\ r=e^{3\theta} \qquad 0\le\theta\le\frac{\pi}{2} \qquad \theta=\frac{\pi}{2}$

$c)\ r=2a\cos\theta \qquad 0\le\theta\le\frac{\pi}{2} \qquad Polar-axis$

$d)\ \begin{cases} x(t)=\sec t \\ y(t)=\tan t \qquad 0\le\theta\le\frac{\pi}{2} \end{cases} \qquad y-axis$

$e)\ \begin{cases} x(t)=4\cos t \\ y(t)=4\sin t \qquad 0\le t\le\frac{\pi}{2} \end{cases} \qquad x-axis$

$f)\ f(x)=\frac{1}{3}x^3 \qquad 0\le x\le 2 \qquad y-aixs$

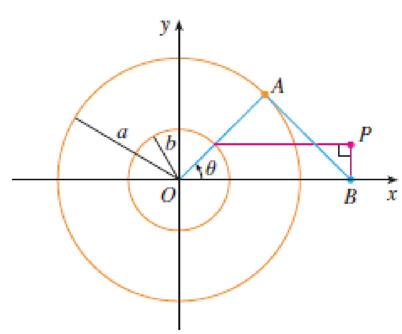
24-Find the area outside of $r^2=Sin2\theta$ and inside of $r=2$ (DO NOT USE GEOMETRY)

25- Use polar coordinate to find the area below line $x+y=2$ from 0 to 2.

26- Write the polar and rectangular equation of a conic with focus at the origin and $e=0.8$, vertex $(1,\ \frac{\pi}{2})$ then graph it.

27- Find the arc length of the graph $x(t)=e^t-t \quad y(t)=4e^{\frac{t}{2}}$ and $0\le t\le\ln 2$.

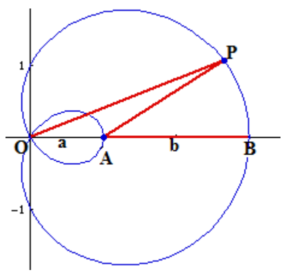
28- If a and b are fixed numbers, find the parametric equations for the curve that consists of all possible positions of the point P in the figure, using the angle θ as the parameter. The line segment AB is tangent to the larger circle.



29-Find the arc length of the graph $x(t)=e^{-t}+t \quad y(t)=4e^{\frac{-t}{2}}$ and $0\le t\le\ln 2$.

30- Find the surface area generated by rotating the $r^2 = \cos(2\theta)$ about the polar axes.

31- Use your pre-Calculus knowledge to find the relation between "a" and "b" such that $3\angle OPA = \angle PAB$ for any point p on the curve $r = a + bCos\theta$



32- A particle launched from height of 30m with speed of 10m/s and angle of 60° in space which offers a gravitational acceleration of -10 $\frac{m}{s^2}$. As it bounces off the ground looses 2/3 of its vertical velocity. Find the distance between the first and the second time that the particle hits the ground.

Use a Graphing Calculator for problem 33 and 35

33- Show the equations for $Epithrochoid\begin{cases} Rose \\ Hypotrochoid \\ Cardioid \\ Nephroid \end{cases}$ and $Hypotrochoid\begin{cases} Hypocycloid \\ Ellipse \\ Deltoid \\ Astroid \end{cases}$

And Go online to graph each one of the above functions (make sure to vary the radiuses)

34- Proof a parametric equation for Prolate and Curtate

35- Given the following parametric equation $\begin{cases} x(t) = rSin(\omega t + \varphi) \\ y(t) = RSin(t) \end{cases}$ Use a graphing Calculator to graph the above equation with specific values $R = r = 1$ otherwise it is given

$\omega = 1, \varphi = 0.5$ $R = r = 1$	$\omega = 1, \varphi = 0.5, R \neq r$	$\omega = 0.5, \varphi = \pi/4$	$\omega = 1, \varphi = 0$
$\omega = 1/2, \varphi = 0$ $R = r = 1$	$\omega = 1/3, \varphi = 0$ $R = r = 1$	$\omega = 1/4, \varphi = 0$	$\omega = 2/5, \varphi = 0$
$\omega = 1/2, \varphi = 0, R = 2, r = 1$	$\omega = 3/5, \varphi = 0, R = 2, r = 1$	$\omega = 3/5, \varphi = 0, R = 2, r = 1$	$\omega = 3/5, \varphi = \pi/3$